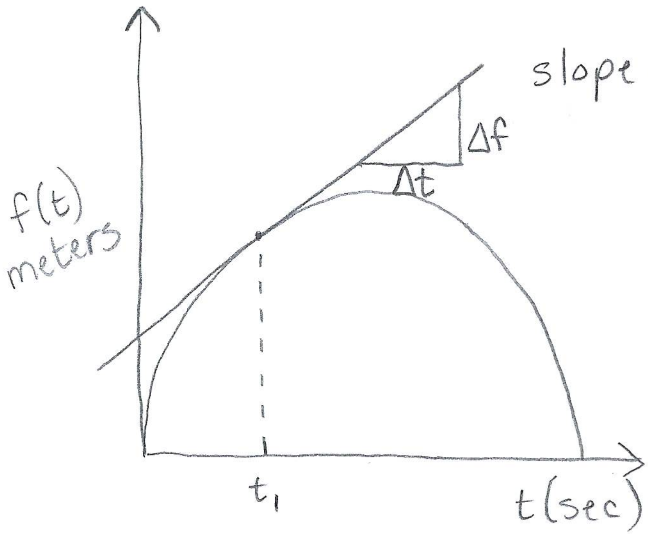
ECE1240

Derivatives in Electric Circuits

Several math based relationships exist for electric circuits that use derivatives. Consider that for a function, $f(t)$, the derivative $\frac{df}{dt}$ is the slope of the line that is tangent to $f(t)$ at time t_1, as shown below:



slope of tangent $= \frac{\text{rise}}{\text{run}} = \frac{\Delta f}{\Delta t}$

There are three typical categories of functions, $f(t)$, used in electrical engineering that may require differentiation (taking a derivative):

1) Polynomials, such as $f(t) = 4t^2 + 6t + 5$
2) Exponentials, such as $f(t) = 4e^{-t}$
3) sin and cos functions, such as $f(t) = \cos(2\pi t)$

Steps for taking the derivative of these are as follows:

1) Consider $f(t) = 4t^2 + 6t + 5$

i) Separate coefficients from variable terms

$$\Rightarrow f(t) = (4 \cdot t^2) + (6 \cdot t) + 5$$

ii) Use the power rule to take the derivative of each term with the variable.
Multiply by power, then reduce power by 1
Coefficients remain out in front.

$$\Rightarrow \frac{d}{dt}(4 \cdot t^2) = 4 \cdot \frac{d}{dt}(t^2) = 4 \cdot (2t^1) = 8t$$

power power-1

iii) Derivative of a constant = 0 because the slope of a constant = 0 $(\Delta f = 0)$.

$$\Rightarrow \frac{d}{dt}(4t^2 + 6t + 5) = \frac{d}{dt}(4t^2) + \frac{d}{dt}(6t) + \frac{d}{dt}(5)$$

$$\Rightarrow \frac{d}{dt}(4t^2 + 6t + 5) = 8t + \left[6 \cdot \frac{d}{dt}(t)\right] + 0$$

$$= 8t + 6 \cdot (1t^0) = 8t + 6$$

power power-1

2) Consider $f(t) = 4e^{-t}$

i) Separate coefficients from exponential terms

$$\Rightarrow f(t) = 4 \cdot e^{-t}$$

ii) The derivative of an exponential is the exponential term multiplied by the coefficient of the variable.

$$\Rightarrow \frac{d}{dt}\left(4e^{-t}\right) = 4 \cdot \frac{d}{dt}\left(e^{-t}\right) = 4 \cdot \underbrace{(-1)}_{\text{variable coefficient}} \underbrace{\left(e^{-t}\right)}_{\text{exponential term}}$$

$$\Rightarrow \frac{d}{dt}\left(4e^{-t}\right) = -4e^{-t}$$

3) Consider $f(t) = 2\cos(2\pi t)$

i) Separate coefficients from trig terms

$$\Rightarrow f(t) = 2 \cdot \cos(2\pi t)$$

ii) Separate coefficient(s) of the variable within the trig function from the variable

$$\Rightarrow f(t) = 2 \cdot \cos\left[(2\pi) \cdot t\right]$$

iii) Use the following to take the derivative of the trig term:

if $f(t) = \sin(at) \Rightarrow \frac{d}{dt}\left(\sin(at)\right) = a \cdot \cos(at)$

if $f(t) = \cos(at) \Rightarrow \frac{d}{dt}\left(\cos(at)\right) = -a \cdot \sin(at)$

$$\Rightarrow \frac{d}{dt}\left(2\cos(2\pi t)\right) = 2 \cdot \frac{d}{dt}\left(\cos(2\pi t)\right)$$

$$\Rightarrow 2 \cdot \frac{d}{dt}\left(\cos(2\pi t)\right) = 2 \cdot (-2\pi)\sin(2\pi t)$$

$$= -4\pi \sin(2\pi t)$$

If $y(t)$ = charge, q, of a circuit

$$\Rightarrow \frac{dy}{dt} = \frac{dq}{dt} = \text{current, } i(t) \text{ amps}$$

If $y(t)$ = energy, W, of a circuit

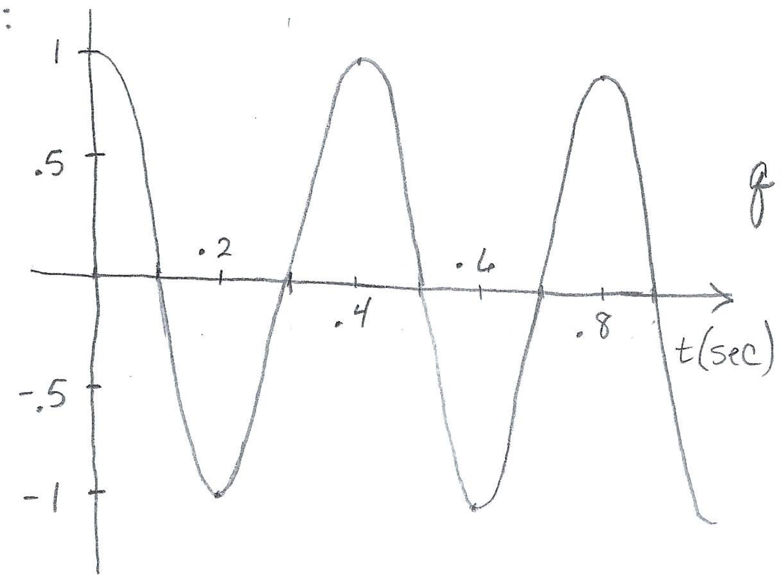
$$\Rightarrow \frac{dy}{dt} = \frac{dW}{dt} = \text{power, } p(t) \text{ Watts}$$

Since power = voltage * current

$$\Rightarrow p(t) = v(t) * i(t) = v(t) * \frac{dq}{dt} = \frac{dW}{dt}$$

$$\Rightarrow v(t) = \frac{\frac{dW}{\cancel{dt}}}{\frac{dq}{\cancel{dt}}} = \frac{dW}{dq}\left(\frac{\text{Joules}}{\text{Coulomb}}\right) \Rightarrow \frac{dW}{dq}\text{(volts)}$$

Consider $q(t)$ for a circuit defined by the graph below:



$q(t) = \cos(5\pi t)$ Coulombs

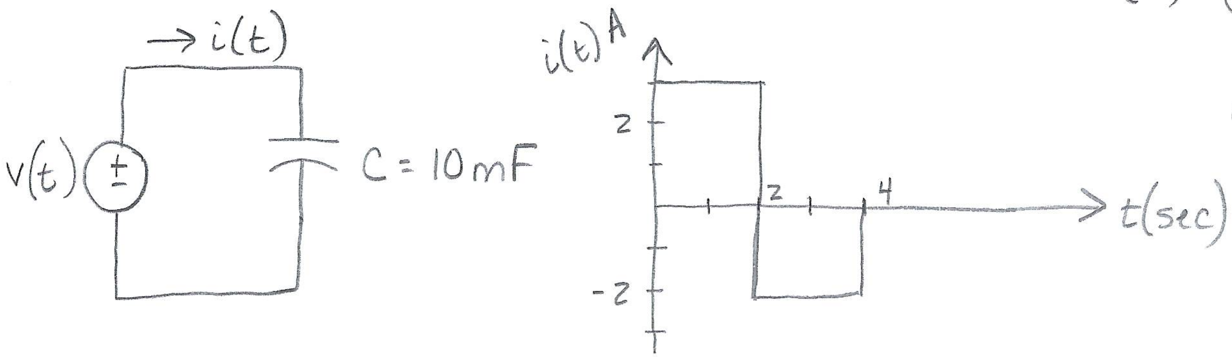
$$\Rightarrow i(t) = \frac{dq}{dt} = \frac{d}{dt}\left[\cos(5\pi t)\right] \text{ amps}$$

From 3iii) on page 3

$$\frac{d}{dt}\left[\cos(at)\right] = -a\sin(at)$$

$$\Rightarrow a = 5\pi \Rightarrow i(t) = -5\pi\sin(5\pi t)$$

Consider the following circuit with current, $i(t)$ shown below and find and sketch $v(t)$. (See Class Module 4 and Chapter 5-2 of textbook)



since $i(t) = \frac{dq}{dt}$ = slope of $q(t)$

$$\Rightarrow i(t) = \frac{dq}{dt} = 3A \text{ for } 0 \le t(sec) \le 2$$

$$i(t) = \frac{dq}{dt} = -2A \text{ for } 2 \le t(sec) \le 4$$

The relationship between $i(t)$ and $v(t)$ in a circuit with capacitance is defined by:

$$i(t) = C\frac{dv}{dt}$$

$$\Rightarrow i(t) = \frac{dq}{dt} = C\frac{dv}{dt}$$

$$\Rightarrow \frac{dv}{dt} = \frac{1}{C}\frac{dq}{dt} \text{ where } C = 10\times10^{-3}F$$

$$\Rightarrow \frac{dv}{dt} = \left(\frac{1}{10\times10^{-3}F}\right)(3A) \quad \text{for } 0 \le t(\text{sec}) \le 2$$

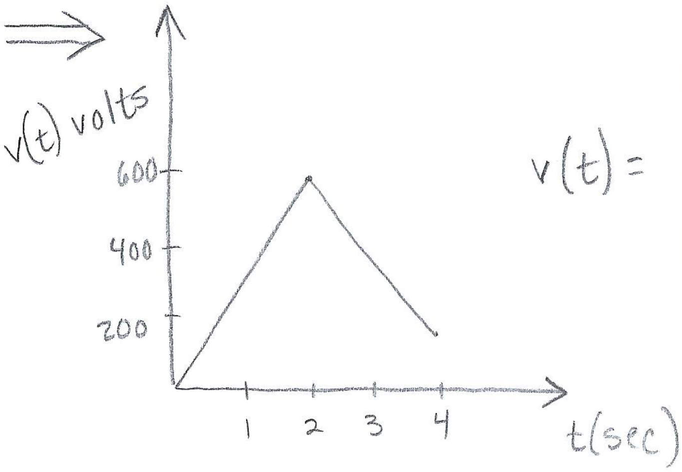
$$= \left(\frac{1}{10\times10^{-3}F}\right)(-2A) \quad \text{for } 2 \le t(\text{sec}) \le 4$$

This corresponds to a voltage, $v(t)$ with slope:

300 for $0 \le t(\text{sec}) \le 2 \Rightarrow v(t) = 300t$ volts

-200 for $2 \le t(\text{sec}) \le 4 \Rightarrow v(t) = -200t$ volts

$\Rightarrow$



$$v(t) = \begin{cases} 300t \text{ volts}, & 0 \le t(\text{sec}) \le 2 \\ -200t \text{ volts}, & 2 \le t(\text{sec}) \le 4 \end{cases}$$

Derivative Practice Problems and Solutions

Find $i(t)$ flowing through a device if $i(t) = \frac{dq}{dt}$ and the charge, $q(t)$, is:

a) $q(t) = (4t^3 + 6t^2 - 1)C$

b) $q(t) = (5e^{-t} - 2e^{-3t})C$

c) $q(t) = 6\sin(12\pi t)C$

Solutions:

a) $i(t) = \frac{dq}{dt} = \frac{d}{dt}(4t^3 + 6t^2 - 1)$

$= 4 \cdot \frac{d}{dt}(t^3) + 6 \cdot \frac{d}{dt}(t^2) + \frac{d}{dt}(-1)$

$\Rightarrow i(t) = 4 \cdot [3 \cdot t^{3-1}] + 6 \cdot [2 \cdot t^{2-1}] + 0$

$= 12t^2 + 12t$

$\Rightarrow i(t) = (12t^2 + 12t)A$

b) $i(t) = \frac{dq}{dt} = \frac{d}{dt}(5e^{-t} - 2e^{-3t})$

$= 5 \cdot \frac{d}{dt}(e^{-t}) - 2 \cdot \frac{d}{dt}(e^{-3t})$

$\Rightarrow i(t) = 5 \cdot e^{-t} \cdot \frac{d}{dt}(-t) - 2 \cdot e^{-3t} \cdot \frac{d}{dt}(-3t)$

$= 5 \cdot e^{-t} \cdot (-1) - 2 \cdot e^{-3t} \cdot (-3)$

$\Rightarrow i(t) = (-5e^{-t} + 6e^{-3t})A = (6e^{-3t} - 5e^{-t})A$

Solutions (cont):

c) $i(t) = \frac{dq}{dt} = \frac{d}{dt}(6\sin 12\pi t)$

$= 6 \cdot \frac{d}{dt}\left[\sin(12\pi t)\right]$

$\Rightarrow i(t) = 6 \cdot \cos(12\pi t) \cdot \frac{d}{dt}(12\pi t)$

$= 6 \cdot \cos(12\pi t) \cdot (12\pi)$

$\Rightarrow i(t) = \left[72\pi \cos(12\pi t)\right] A$