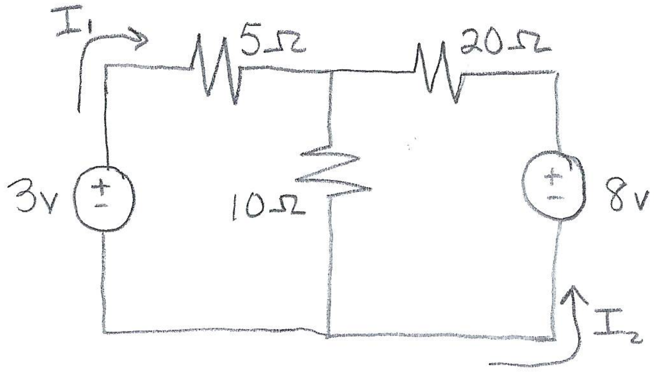
ECE1240

Matrix Math

Systems of equations can be solved using matrices. We'll start with a simple example using the following two loop circuit:



To find I_1 + I_2, we use KVL (Kirchoff's Voltage Law) to obtain the following equations:

$$\Longrightarrow (5\Omega + 10\Omega)(I_1) + (10\Omega)(I_2) = 3v$$
$$(10\Omega)(I_1) + (20\Omega + 10\Omega)(I_2) = 8v$$

$$\Longrightarrow (15\Omega)(I_1) + (10\Omega)(I_2) = 3v$$
$$(10\Omega)(I_1) + (30\Omega)(I_2) = 8v$$

elements of A

elements of b

This can be solved using a 2x2 matrix of the form:

$Ax = b$ where

A = coefficient matrix = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\Longrightarrow A = \begin{bmatrix} 15 & 10 \\ 10 & 30 \end{bmatrix}$ for this problem

x = unknowns we're solving for

$\Rightarrow x = \begin{Bmatrix} I_1 \\ I_2 \end{Bmatrix}$ for this problem

b = right-hand side values

$\Rightarrow b = \begin{Bmatrix} 3 \\ 8 \end{Bmatrix}$ for this problem

To solve for x in $Ax=b$, multiply both sides by inverse of A, written as A^{-1}

$\Rightarrow A^{-1} \cdot Ax = A^{-1} \cdot b$

$\Rightarrow A^{-1} \cdot A = I$, identity matrix

$\Rightarrow (I)(x) = A^{-1} \cdot b$

Since $(I)(x) = x \Rightarrow x = A^{-1} \cdot b$

For $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $A^{-1} = \dfrac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Now to solve $x = A^{-1} \cdot b$

$$\Rightarrow x = \frac{1}{\underset{a}{(15)}\underset{d}{(30)} - \underset{b}{(10)}\underset{c}{(10)}} \begin{bmatrix} 30 & -10 \\ -10 & 15 \end{bmatrix} \cdot b$$

$$= \frac{1}{450-100} \begin{bmatrix} 30 & -10 \\ -10 & 15 \end{bmatrix} \cdot \begin{Bmatrix} 3 \\ 8 \end{Bmatrix}$$

$$= \frac{1}{350} \begin{bmatrix} 30 & -10 \\ -10 & 15 \end{bmatrix} \cdot \begin{Bmatrix} 3 \\ 8 \end{Bmatrix}$$

$$= \begin{bmatrix} 3/35 & -1/35 \\ -1/35 & 3/70 \end{bmatrix} \cdot \begin{Bmatrix} 3 \\ 8 \end{Bmatrix}$$

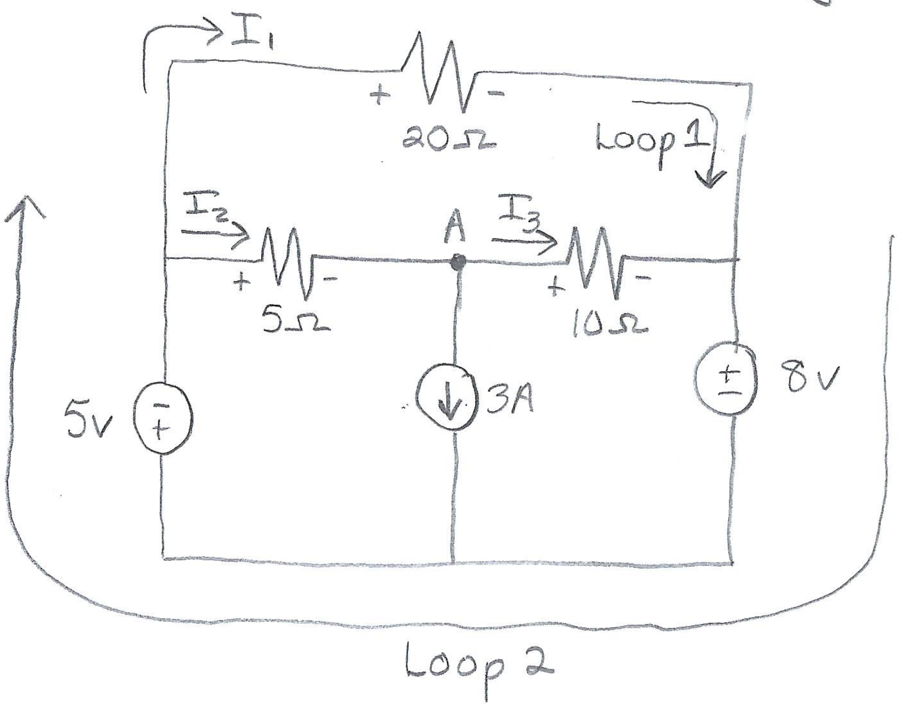
$$\Rightarrow X = \begin{Bmatrix} (3/35)(3) + (-1/35)(8) \\ (-1/35)(3) + (3/70)(8) \end{Bmatrix} = \begin{Bmatrix} 1/35 \\ 9/35 \end{Bmatrix} = \begin{Bmatrix} I_1 \\ I_2 \end{Bmatrix}$$

$$\Rightarrow I_1 = 1/35 = 0.029\ A$$
$$I_2 = 9/35 = 0.257\ A$$

Plug these values into the original equation to check your answer

$$(15\Omega)(I_1) + (10\Omega)(I_2) = 3V$$
$$(15\Omega)(0.029A) + (10\Omega)(0.257A) = 3.00V \ \checkmark$$

Larger systems of equations require larger matrices. These are more easily solved using a calculator or Matlab. Consider the following circuit:



To solve for $I_1, I_2, + I_3$, use KVL + KCL to write equations:

Loop 1: $(20\Omega)(I_1) - (10\Omega)(I_3) - (5\Omega)(I_2) = 0$ (KVL)

Loop 2: $(20\Omega)(I_1) + 8V + 5V = 0$ (KVL)

Node A: $I_2 - 3A - I_3 = 0$ (KCL)

$\Rightarrow$ 3 equations, 3 unknowns

Group unknowns in corresponding columns

$$\Rightarrow \begin{aligned} 20I_1 - 5I_2 - 10I_3 &= 0 \\ 20I_1 \qquad\qquad\qquad &= -8-5 = -13 \\ I_2 - I_3 &= 3 \end{aligned}$$

Now form the matrices:

$$\underbrace{\overset{A}{\begin{bmatrix} 20 & -5 & -10 \\ 20 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix}}}_{\text{coefficients}} \underbrace{\overset{X}{\begin{Bmatrix} I_1 \\ I_2 \\ I_3 \end{Bmatrix}}}_{\text{unknowns}} = \underbrace{\overset{b}{\begin{Bmatrix} 0 \\ -13 \\ 3 \end{Bmatrix}}}_{\text{right hand side}}$$

As before:

$$X = A^{-1} \cdot b$$

Use Matlab to create a new script file to calculate:

Matlab code

```
1) Clear all;
2) % Define coefficient matrix A in []
3) % Separate elements in a row by commas
4) % Separate rows by semicolons
5) A = [20,-5,-10;20,0,0;0,1,-1];
6) % Define right-hand side matrix b in []
7) b = [0,-13,3];
8) %. Solve for I1, I2, I3
9) % Leave out the semicolon for result
10) % to show in Matlab window
11) I = A^-1*b
12) % This produces the following display:

I =
    -0.6500
     1.1333
    -1.8666
```

$\Rightarrow I_1 = -0.65, \; I_2 = 1.13, \; I_3 = -1.86$

Plug these values into Loop 1 equation to check:

$$(20)(-0.65) - (10)(-1.867) - (5)(1.133) = 0.005 \approx 0 \checkmark$$

Using the signs (+/-) for calculated values, we can redraw the circuit accurately: