Homework 1
Spring 2019 Solutions

Exercise 1. (10 pts)

(a) $(72)_{10} = (1001000)_2$

(b) $(34)_{10} = (100010)_2$

(c) $(712)_{10} = (1011001000)_2$

(d) $(171)_{10} = (10101011)_2$

(e) $(888)_{10} = (1101111000)_2$

Exercise 2. (10 pts)

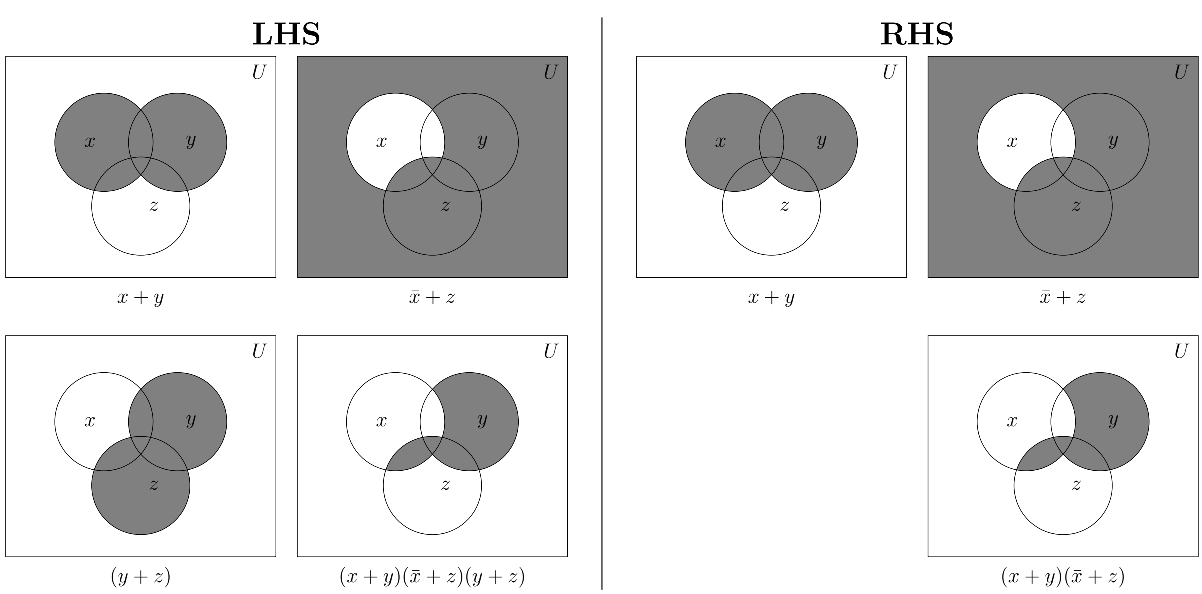
(a) $(1100)_2 = (12)_{10}$

(b) $(10101)_2 = (21)_{10}$

(c) $(101101)_2 = (45)_{10}$

(d) $(1100111000)_2 = (824)_{10}$

Exercise 3. (10 pts)



LHS: $x+y$, $\bar{x}+z$, $(y+z)$, $(x+y)(\bar{x}+z)(y+z)$

RHS: $x+y$, $\bar{x}+z$, $(x+y)(\bar{x}+z)$

Since the venn diagrams in bottom right corner are the same, the statement is proved.

Exercise 4. (10 pts)

$$
\begin{aligned}
f(x_1, x_2, x_3, x_4) &= x_1\bar{x_3}x_4 + \bar{x_2}\bar{x_3}\bar{x_4} + \bar{x_1}x_2\bar{x_3}x_4 \\
&= \bar{x_2}\bar{x_3}\bar{x_4} + x_1\bar{x_3}x_4 + \bar{x_1}x_2\bar{x_3}x_4 && \text{10b (Commutative)} \\
&= \bar{x_3}(\bar{x_2}\bar{x_4} + x_1x_4 + \bar{x_1}x_2x_4) && \text{12a (Distributive)} \\
&= \bar{x_3}(\bar{x_2}\bar{x_4} + x_4(x_1 + \bar{x_1}x_2)) && \text{12a (Distributive)} \\
&= \bar{x_3}(\bar{x_2}\bar{x_4} + x_4(x_1 + x_2)) && \text{16a (DeMorgan's)} \\
&= \bar{x_3}(\bar{x_2}\bar{x_4} + x_1x_4 + x_2x_4) && \text{12a (Distributive)} \\
&= \bar{x_2}\bar{x_3}\bar{x_4} + x_1\bar{x_3}x_4 + x_2\bar{x_3}x_4 && \text{12a (Distributive)}
\end{aligned}
$$

Exercise 5. (20 pts)

Minimum-cost sum-of-products (SOP) form is found by grouping together 1's in the Karnaugh map. Groups may also include don't cares if they help with reduction.

Following the setup for a Karnaugh map as seen on p.84 of the textbook in Fig. 2.53, treat a_0 as the MSB (Most Significant Bit) and b_1 as the LSB (Least Significant Bit). Thus, minterm 1 is when $a_0a_1b_0b_1 = (0001)_2$, and minterm 8 is when $a_0a_1b_0b_1 = (1000)_2$.

b_0b_1 \ a_0a_1	00	01	11	10
00	**m0**	**m4**	**m12**	**m8**
01	**m1**	**m5**	**m13**	**m9**
11	**m3**	**m7**	**m15**	**m11**
10	**m2**	**m6**	**m14**	**m10**

$$\textbf{sum}_1(a_0, a_1, b_0, b_1) \quad = \quad \sum m(1,3,4,6,9,10,12) + D(5,7,11,13,14,15)$$

b_0b_1 \ a_0a_1	00	01	11	10
00	**0**	**1**	**1**	**0**
01	**1**	**d**	**d**	**1**
11	**1**	**d**	**d**	**d**
10	**0**	**1**	**d**	**1**

Minimum-cost SOP: $sum_1(a_0, a_1, b_0, b_1) = a_1 + b_1 + a_0b_0$

$$\mathbf{sum}_0(a_0, a_1, b_0, b_1) \quad = \quad \sum m(2, 3, 6, 8, 9, 12) + D(5, 7, 11, 13, 14, 15)$$

b_0b_1 \ a_0a_1	00	01	11	10
00	**0**	**0**	**1**	**1**
01	**0**	**d**	**d**	**1**
11	**1**	**d**	**d**	**d**
10	**1**	**1**	**d**	**0**

Minimum-cost SOP: $sum_0(a_0, a_1, b_0, b_1) = a_0\bar{b_0} + \bar{a}_0b_0$

$$\mathbf{diff}_1(a_0, a_1, b_0, b_1) \quad = \quad \sum m(6, 7, 9, 13) + D(10, 11, 14, 15)$$

b_0b_1 \ a_0a_1	00	01	11	10
00	**0**	**0**	**0**	**0**
01	**0**	**0**	**1**	**1**
11	**0**	**1**	**d**	**d**
10	**0**	**1**	**d**	**d**

Minimum-cost SOP: $diff_1(a_0, a_1, b_0, b_1) = a_0b_1 + a_1b_0$

$$\textbf{diff}_0(a_0, a_1, b_0, b_1) \quad = \quad \sum m(5, 7, 13) + D(10, 11, 14, 15)$$

b_0b_1 \ a_0a_1	00	01	11	10
00	**0**	**0**	**0**	**0**
01	**0**	**1**	**1**	**0**
11	**0**	**1**	**d**	**d**
10	**0**	**0**	**d**	**d**

Minimum-cost SOP: $diff_0(a_0, a_1, b_0, b_1) = a_1b_1$

Exercise 6. (10 pts)

Minimum cost product of sums (POS) form is found by grouping together 0's in the Karnaugh map. Groups may also include don't cares if they help with reduction.

$$\textbf{overflow\_sum}(a_0, a_1, b_0, b_1) \quad = \quad \sum m(5, 7, 11, 13, 14, 15)$$

b_0b_1 \ a_0a_1	00	01	11	10
00	**0**	**0**	**0**	**0**
01	**0**	**1**	**1**	**0**
11	**0**	**1**	**1**	**1**
10	**0**	**0**	**1**	**0**

Minimum-cost POS:

$$\textbf{overflow\_sum}(a_0, a_1, b_0, b_1) = (b_0 + b_1)(a_0 + a_1)(a_1 + b_0)(a_0 + b_1)(a_1 + b_1)$$

$$\textbf{underflow\_diff}(a_0, a_1, b_0, b_1) = \sum m(4, 6, 8, 12, 13, 14, 15)$$

b_0b_1 \ a_0a_1	00	01	11	10
00	**0**	**1**	**1**	**1**
01	**0**	**0**	**1**	**0**
11	**0**	**0**	**1**	**0**
10	**0**	**1**	**1**	**0**

Minimum-cost POS: $underflow\_diff(a_0, a_1, b_0, b_1) = (a_0 + a_1)(a_0 + \bar{b}_1)(a_1 + \bar{b}_1)(a_1 + \bar{b}_0)$

Exercise 7. (30 pts)

This problem is meant to be a pre-lab for Lab 1, and therefore, no Verilog solution is given to avoid explicit copying.