Asynchronous Circuit Design

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Lecture 7: Timed Circuits
Chapter 7

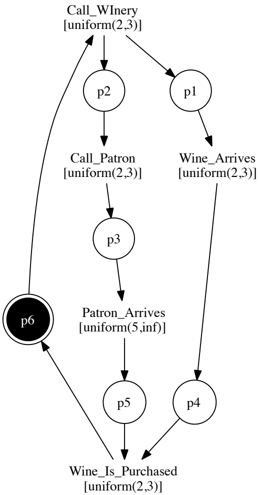
Timed Circuits

- Previous methods only use limited knowledge of delays.
- Very robust systems, but extremely conservative.
- Large functional units do not have zero delay.
- Gates and wires do not have an infinite delay.
- Timing analysis can identify additional unreachable states.
- These unreachable states are additional don’t cares.
- *Timed circuits* use this information to optimize the design.

A Simple Example

- Shopkeeper actively calls winery and patron.
- Calls the patron immediately after calling the winery without waiting for the wine to arrive.
- The shopkeeper does the following:
  - Calls the winery,
  - Calls the patron,
  - Peers out the window until he sees both the wine delivery boy and the patron,
  - Lets them in, and
  - Completes the sale.

Timing Relationships



Timed States

- There is a timer t_i associated with each arc in the graph.
- A *timed state* is an *untimed state* and value of all active *timers*.

$$(\{p_6\}, t6 = 0)$$

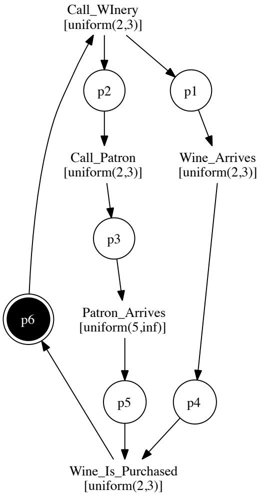
- A timer is allowed to advance by any amount less than its upper bound resulting in a new timed state.

$$(\{p_6\}, t6 = 1.1)$$
$$(\{p_6\}, t6 = 2.22)$$
$$(\{p_6\}, t6 = 2.71828182846)$$

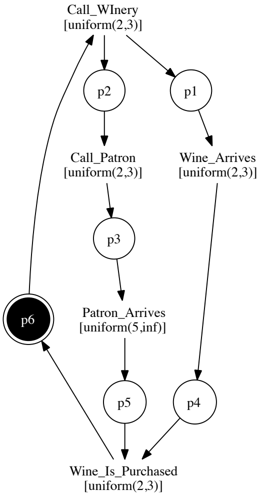
Timing Sequences

- When a timer reaches its lower bound, it becomes *satisfied*.
- When a timer reaches its upper bound, it becomes *expired*.
- An event enabled by a single rule must happen sometime after its timer becomes satisfied and before it becomes expired.
- When an event is enabled by multiple rules, it must happen after all of its rules are satisfied, but before all of its rules are expired.
- Extend the notion of allowed sequences to timed states paired with the time of the state transition.
- State transition can be either time advancement or a change in the untimed state.

Example: Timing Sequence

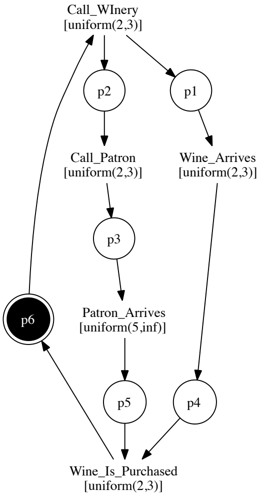


Example: Timing Sequence



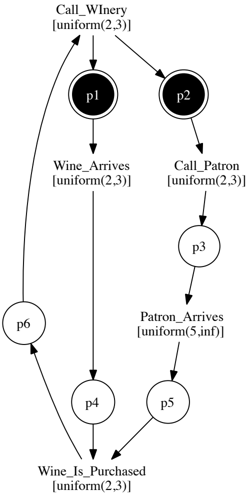
$$(\ ([\{p_6\},\ t_6 = 0],\ 0),$$

Example: Timing Sequence



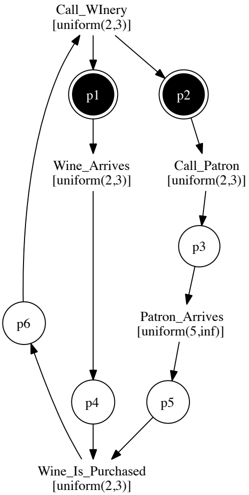
$$(([\{p_6\}, t_6 = 0], 0),$$
$$([\{p_6\}, t_6 = 2.22], 2.22),$$

Example: Timing Sequence



$$
\begin{aligned}
(\ & ([\{p_6\},\ t_6 = 0],\ 0), \\
& ([\{p_6\},\ t_6 = 2.22],\ 2.22), \\
& ([\{p_1, p_2\},\ t_1 = t_2 = 0],\ 2.22),
\end{aligned}
$$

Example: Timing Sequence



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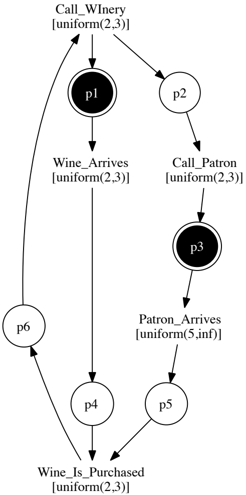
& ([\{p_1, p_2\},\ t_1 = t_2 = 2.1],\ 4.32),
\end{aligned}
$$

Example: Timing Sequence



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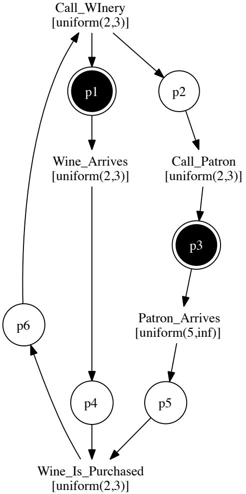
& ([\{p_1, p_2\},\ t_1 = t_2 = 2.1],\ 4.32), \\
& ([\{p_1, p_3\},\ t_1 = 2.1,\ t_3 = 0],\ 4.32),
\end{aligned}
$$

Example: Timing Sequence



$$
\begin{aligned}
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&([\{p_6\},\ t_6 = 2.22],\ 2.22),\\
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&([\{p_1, p_2\},\ t_1 = t_2 = 2.1],\ 4.32),\\
&([\{p_1, p_3\},\ t_1 = 2.1,\ t_3 = 0],\ 4.32),\\
&\ldots
\end{aligned}
$$

Timed State Space Exploration

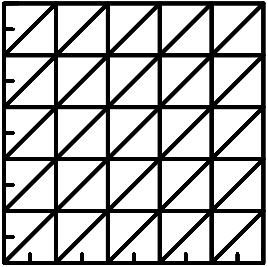
- Since time can take on any real value, there is an uncountably infinite number of timed states and timed allowed sequences.
- Must either group timed states into finite number of equivalence classes or restrict the values of the timers.
- Several possible methods for *timed state space exploration*:
  - Region method
  - Discrete-time method
  - Zone method
  - POSET method

Regions

- A *region* is described by the integer component of each timer and the relationship between the fractional components.
- $f(t_1) = f(t_2) = 0$: region is a point.
- $f(t_1) = 0$ and $f(t_2) > 0$: region is a vertical line segment.
- $f(t_1) > 0$ and $f(t_2) = 0$: region is a horizontal line segment.
- $f(t_1) = f(t_2) > 0$: region is a diagonal line segment.
- $f(t_1) > f(t_2) > 0$: region is an lower triangle.
- $f(t_2) > f(t_1) > 0$: region is an upper triangle.

Possible Timed States Using Regions

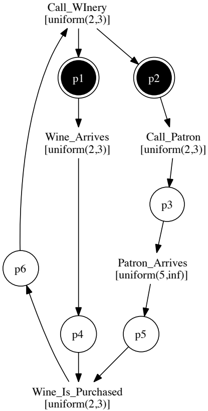
$t_2 \langle 0, 5 \rangle$

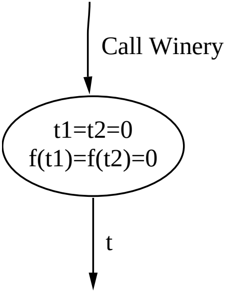


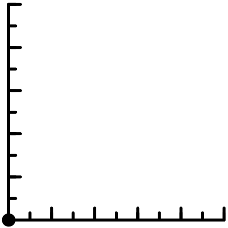
$t_1 \langle 0, 5 \rangle$

171 distinct timed states

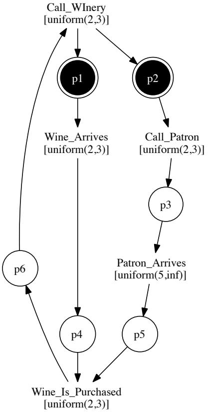
Timed Sequence Using Regions

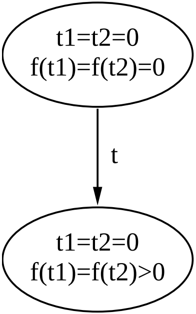


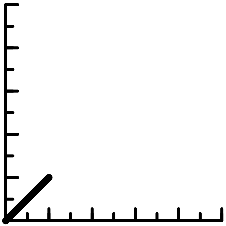




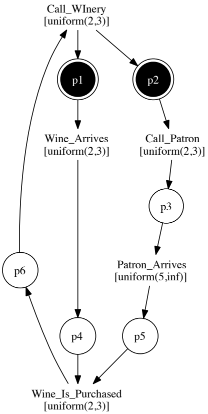
Timed Sequence Using Regions

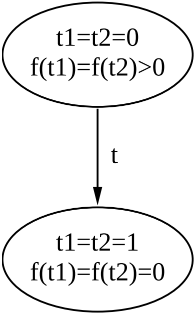


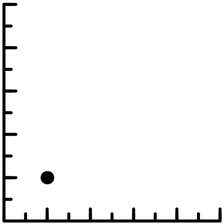




Timed Sequence Using Regions







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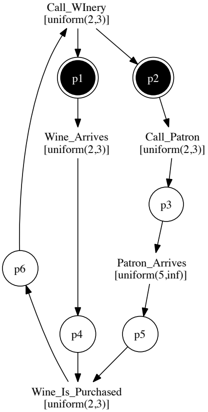

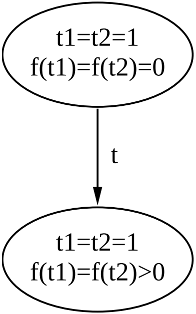

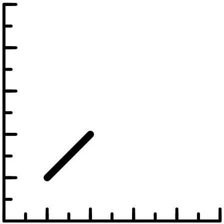

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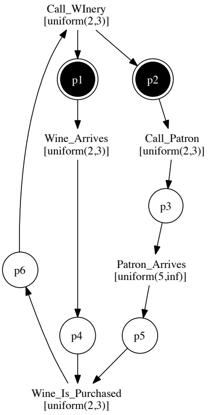

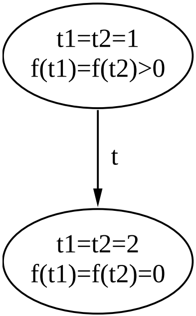

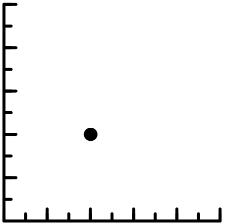

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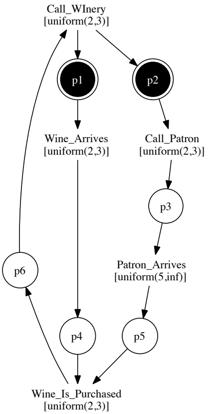

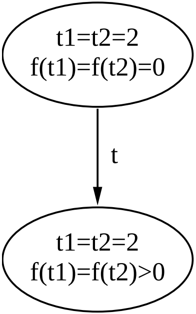

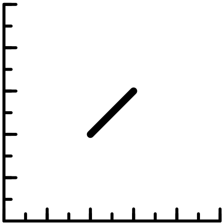

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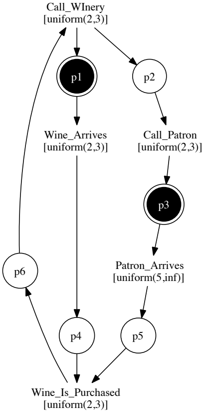

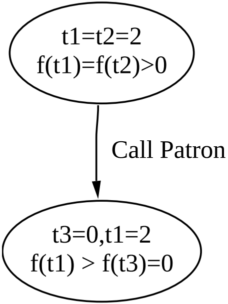

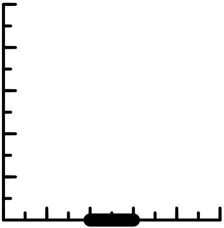

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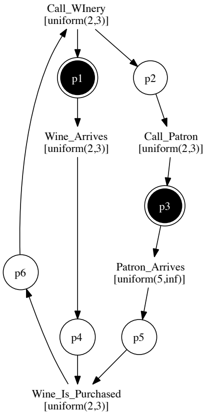

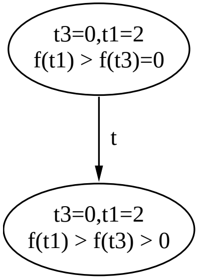

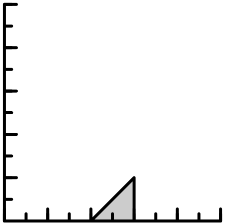

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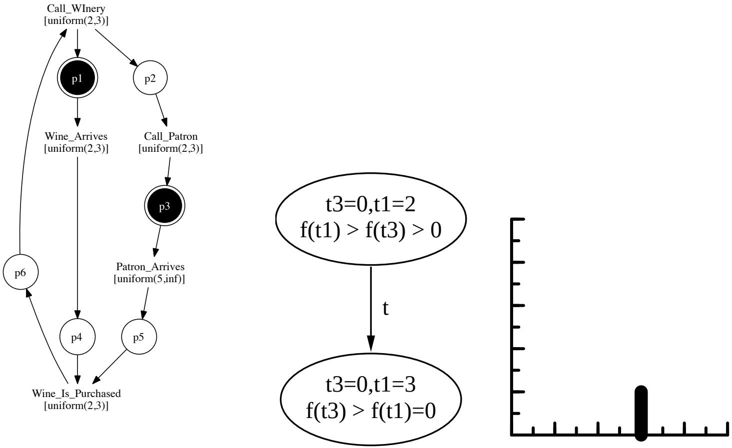

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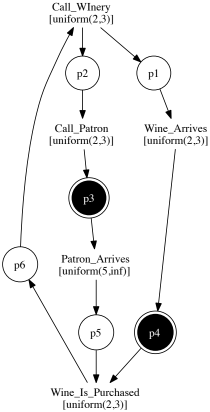
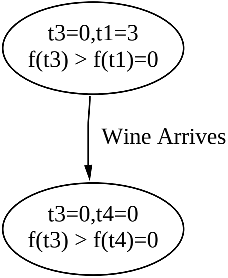
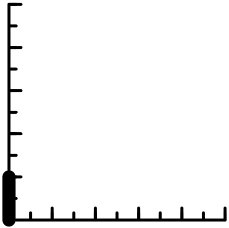
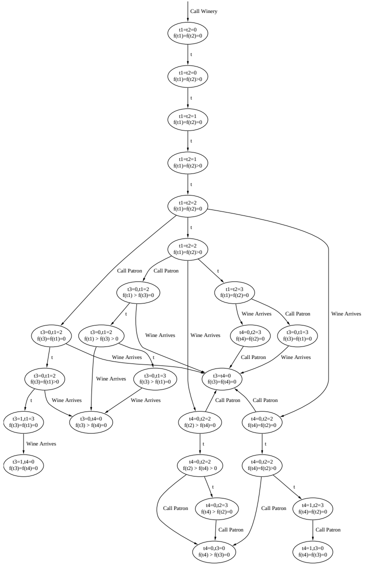






Timed State Space Using Regions



Timed State Space Explosion

- Requires 26 timed states to represent all the timing relationships for only 4 untimed states.
- Worst-case complexity is:

$$|S| \frac{n!}{ln2} \left(\frac{k}{ln2} \right)^n 4^{1/k}$$

where S is number of untimed states, n is the number of rules enabled concurrently, and k is maximum timing constraint.

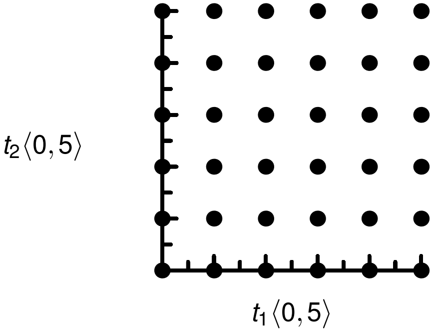
Discrete-Time

- For timed labeled Petri-nets, all timing requirements are of the form $\leq$ or $\geq$, since timing bounds are inclusive.
- In this case, fractional components are not necessary.
- Only need to track *discrete-time* states.
- Worst-case complexity is now:

$$|S|(k+1)^n$$

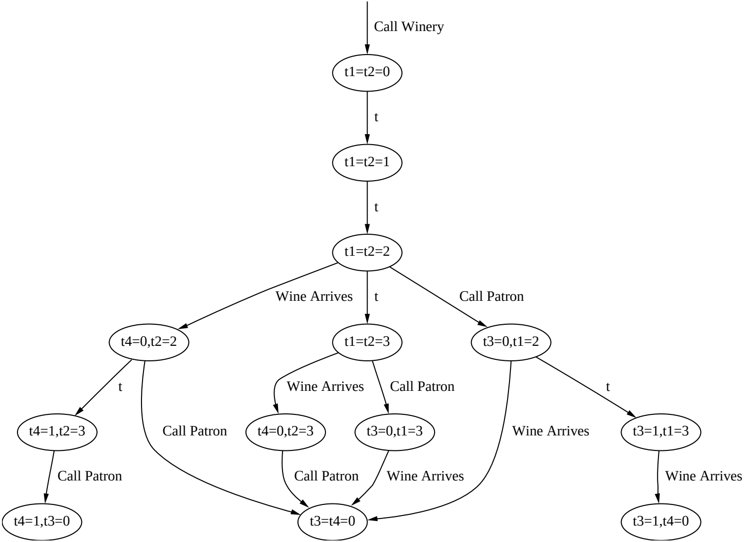
- Reduction by a factor of more than $n!$.

Possible Timed States Using Discrete-Time



36 distinct timed states

Timed State Space Using Discrete-Time

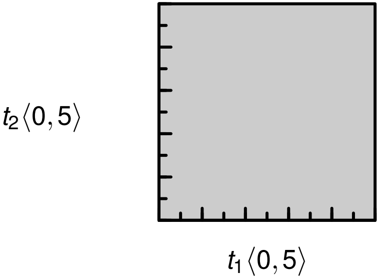


Timed State Space Explosion Again

- Unfortunately, the discrete-time technique is still exponential in the number of concurrent timers and size of the timing bounds.
- Changing each timing bound of $[2, 3]$ to $[19, 31]$ and $[5, \mathit{inf}]$ to $[53, \mathit{inf}]$, number of timed states goes from 69 to more than 3000.
- Changing each timing bound of $[2, 3]$ to $[191, 311]$ and $[5, \mathit{inf}]$ to $[531, \mathit{inf}]$, number of timed states goes to over 300,000!

Zones

- Another approach is to use convex polygons, called *zones*, to represent equivalence classes of timed states.
- One zone is representing 171 regions or 36 discrete-time states.



Representing Zones using Linear Inequalities

- Convex polygons can be represented using linear inequalities.
- Introduce a dummy timer t_0 which always takes the value 0.
- For each pair of timers, introduce inequality of the form:

$$t_j - t_i \quad \leq \quad m_{ij}$$

- Example:

$$\begin{array}{ll}
t_0 - t_0 \leq 0 & \\
t_1 - t_0 \leq 5 & t_2 - t_1 \leq 5 \\
t_2 - t_0 \leq 5 & t_0 - t_2 \leq 0 \\
t_0 - t_1 \leq 0 & t_1 - t_2 \leq 5 \\
t_1 - t_1 \leq 0 & t_2 - t_2 \leq 0
\end{array}$$

- Set of inequalites can be collected into a data structure called a *difference bound matrix* (DBM).
- The difference bound matrix for this example is shown below:

$$\begin{array}{c|ccc|} & t_0 & t_1 & t_2 \\ t_0 & 0 & 5 & 5 \\ t_1 & 0 & 0 & 5 \\ t_2 & 0 & 5 & 0 \end{array}$$

Recanonicalization

- Many DBMs represent the same zone.
- Need unique DBM representation to determine when a zone has been seen before during the depth first search.
- Each zone has a canonical DBM representation when all entries are minimal.

Recanonicalization Example

$$
\begin{array}{c|ccc|}
 & t_0 & t_1 & t_2 \\
\hline
t_0 & 0 & 5 & 5 \\
t_1 & 0 & 0 & 7 \\
t_2 & 0 & 5 & 0
\end{array}
\qquad
\begin{array}{rcl}
t_2 - t_1 & \leq & 7 \\
t_2 - t_0 & \leq & 5 \\
t_0 - t_1 & \leq & 0
\end{array}
$$

- Add last two equations to get:

$$t_2 - t_1 \quad \leq \quad 5$$

$$
\begin{array}{c|ccc|}
 & t_0 & t_1 & t_2 \\
\hline
t_0 & 0 & 5 & 5 \\
t_1 & 0 & 0 & 5 \\
t_2 & 0 & 5 & 0
\end{array}
$$

DBM as Digraph

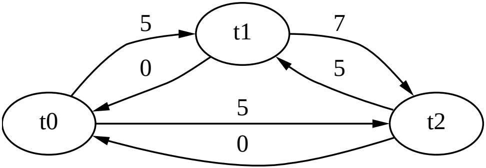
- Recanonicalization equivalent to all pairs shortest path problem.
- Create a labeled digraph where:
  - There is a vertex for each timer t_i,
  - An arc from t_i to t_j for each linear inequality of the form $t_j - t_i \leq m_{ij}$ when $i \neq j$.
  - Each arc is labeled by m_{ij}.

Floyd's Algorithm

recanonicalization (M)
  for $k = 1$ **to** n
    for $i = 1$ **to** n
      for $j = 1$ **to** n
        if $(m_{ij} > m_{ik} + m_{kj})$ **then**
          $m_{ij} = m_{ik} + m_{kj}$**;**

Floyd's Algorithm Example

$$\begin{array}{c|ccc|} & t_0 & t_1 & t_2 \\ \hline t_0 & 0 & 5 & 5 \\ t_1 & 0 & 0 & 7 \\ t_2 & 0 & 5 & 0 \end{array}$$



Zone Creation

- After a rule fires:
  - `Restrict` to reflect minimum firing time.
  - `Reconönicalize`
  - `Project` out information on rule that fired.
  - `Extend` matrix with new rows and columns for new rules.
  - `Advance time`
  - `Recononicalize`

Restrict

- Example: firing of a rule $r_k = \langle e_k, f_k, l_k, u_k \rangle$ where
  - e_k is the enabling transition,
  - f_k is the enabled transition,
  - l_k is the lower bound of the cooresponding timer t_k, and
  - u_k is the upper bound on the timer.
- Constrain DBM to indicate rule has reached its lower bound.
- $t_0 - t_k \leq -l_k$, so set m_{k0} to $-l_k$.
- DBM may no longer be maximally tight.
- Recanonicalize DBM using Floyd's algorithm.

Project

- Remove the row and column cooresponding to t_k.
- If rule firing causes an transition, new rules may be enabled.
- For newly enabled rules, introduce a new timer t_l with a row and column in the DBM.
- Initialize m_{l0} and m_{0l} to 0.
- Initialize each m_{lj} to m_{0j}.
- Initialize each m_{il} to m_{i0}.

Advance Time

- Set all timers to their upper bound (i.e., $m_{0j} = u_j$).
- Recanonicalize the DBM using Floyd's algorithm.

Update Zone

```
update_zone(M, r_j, event_fired, R_en, R_new)
  if m_j0 > -l_j then m_j0 = -l_j
  recanonicalize(M)
  project(M, r_j)
  if (event_fired) then
    foreach r_i ∈ R_new
      m_i0 = m_0i = 0
      foreach r_k ∈ R_new
        m_ik = m_ki = 0
      foreach r_k ∈ (R_en − R_new)
        m_ik = m_0k
        m_ki = m_k0
  foreach r_i ∈ R_en
    m_0i = u_i
  recanonicalize(M)
  normalize(M, R_en)
```

Normalize

```
normalize(M, R_en)
  foreach r_i ∈ R_en
    if (m_i0 < -premax(r_i)) then
      foreach r_j ∈ R_en
        m_ij = m_ij - (m_i0 + premax(r_i))
        m_ji = m_ji + (m_i0 + premax(r_i))
  foreach r_i ∈ R_en
    if (m_0i > premax(r_i)) then
      m_0i = max_j(min(m_0j, premax(r_j)) - m_ij)
  recanonicalize(M)
```

normalize(M, R_{en})

  foreach $r_i \in R_{en}$

    if $(m_{i0} < -premax(r_i))$ **then**

      foreach $r_j \in R_{en}$

        $m_{ij} = m_{ij} - (m_{i0} + premax(r_i))$

        $m_{ji} = m_{ji} + (m_{i0} + premax(r_i))$

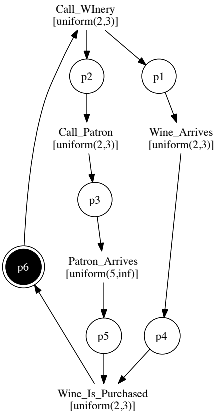
  foreach $r_i \in R_{en}$

    if $(m_{0i} > premax(r_i))$ **then**

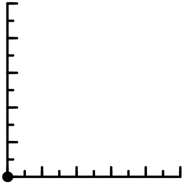
      $m_{0i} = \max_j(\min(m_{0j}, premax(r_j)) - m_{ij})$

  recanonicalize(M)

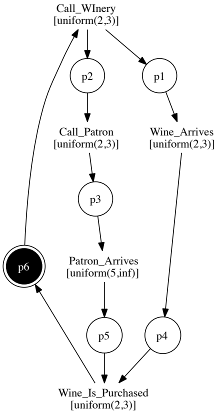
Initial Zone



Initial $\begin{array}{c|cc|} & t_0 & t_6 \\ t_0 & 0 & 0 \\ t_6 & 0 & 0 \end{array}$

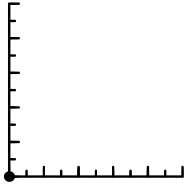


Initial Zone



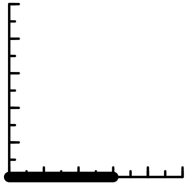
Initial

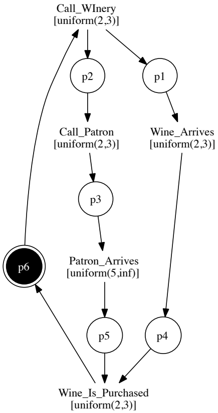
$$\begin{array}{c|cc|} & t_0 & t_6 \\ t_0 & 0 & 0 \\ t_6 & 0 & 0 \end{array}$$



AdvTime/
Recanon/
Norm.

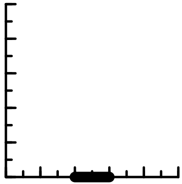
$$\begin{array}{c|cc|} & t_0 & t_6 \\ t_0 & 0 & 3 \\ t_6 & 0 & 0 \end{array}$$



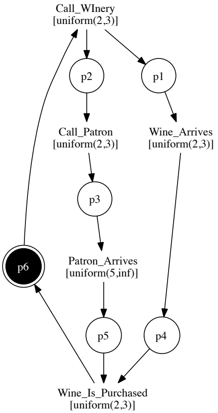


Restrict/
Recanon

$$\begin{array}{c|cc|} & t_0 & t_6 \\ t_0 & 0 & 3 \\ t_6 & -2 & 0 \end{array}$$

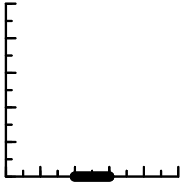


Zone After Winery Called



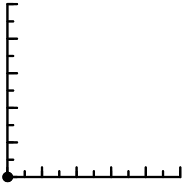
Restrict/
Recanon

$$\begin{array}{c|cc|} & t_0 & t_6 \\ \hline t_0 & 0 & 3 \\ t_6 & -2 & 0 \end{array}$$

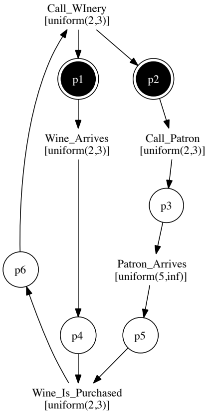


Project

$$\begin{array}{c|c|} & t_0 \\ \hline t_0 & 0 \end{array}$$

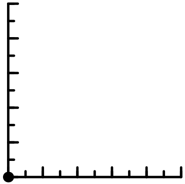


Zone After Winery Called



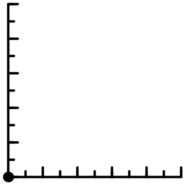
Project

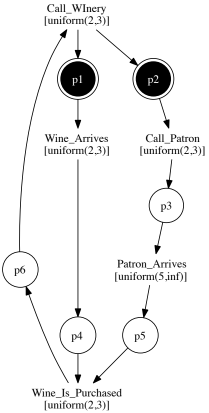
$$\begin{array}{c|c|} & t_0 \\ t_0 & 0 \end{array}$$



Extend

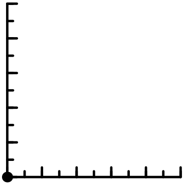
$$\begin{array}{c|ccc|} & t_0 & t_1 & t_2 \\ t_0 & 0 & 0 & 0 \\ t_1 & 0 & 0 & 0 \\ t_2 & 0 & 0 & 0 \end{array}$$





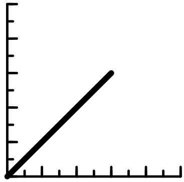
Extend

$$\begin{array}{c|ccc|} & t_0 & t_1 & t_2 \\ t_0 & 0 & 0 & 0 \\ t_1 & 0 & 0 & 0 \\ t_2 & 0 & 0 & 0 \end{array}$$

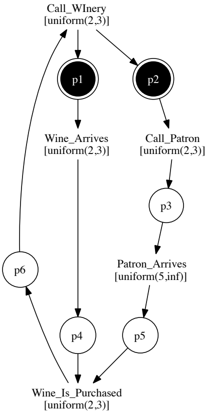


AdvTime/
Recanon/
Norm.

$$\begin{array}{c|ccc|} & t_0 & t_1 & t_2 \\ t_0 & 0 & 3 & 3 \\ t_1 & 0 & 0 & 0 \\ t_2 & 0 & 0 & 0 \end{array}$$

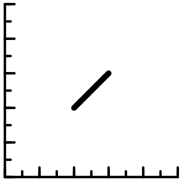


Zone After Wine Arrives

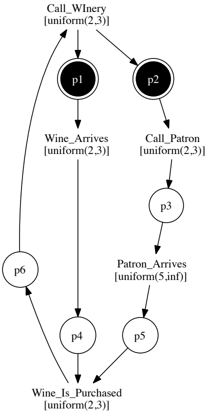


Restrict

$$\begin{array}{c|ccc|} & t_0 & t_1 & t_2 \\ t_0 & 0 & 3 & 3 \\ t_1 & -2 & 0 & 0 \\ t_2 & 0 & 0 & 0 \end{array}$$

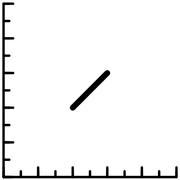


Zone After Wine Arrives



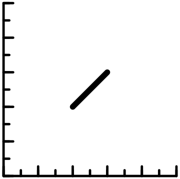
Restrict

$$\begin{array}{c} \\ t_0 \\ t_1 \\ t_2 \end{array} \begin{array}{c} \begin{array}{ccc} t_0 & t_1 & t_2 \end{array} \\ \left|\begin{array}{ccc} 0 & 3 & 3 \\ -2 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right| \end{array}$$

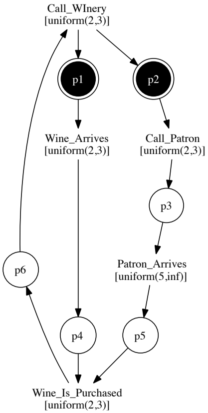


Recanon

$$\begin{array}{c} \\ t_0 \\ t_1 \\ t_2 \end{array} \begin{array}{c} \begin{array}{ccc} t_0 & t_1 & t_2 \end{array} \\ \left|\begin{array}{ccc} 0 & 3 & 3 \\ -2 & 0 & 0 \\ -2 & 0 & 0 \end{array}\right| \end{array}$$

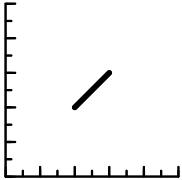


Zone After Wine Arrives



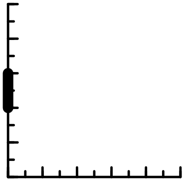
Recanon

$$\begin{array}{c|ccc|} & t_0 & t_1 & t_2 \\ t_0 & 0 & 3 & 3 \\ t_1 & -2 & 0 & 0 \\ t_2 & -2 & 0 & 0 \end{array}$$

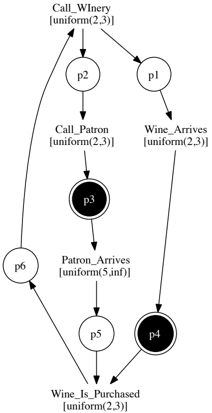


Project

$$\begin{array}{c|cc|} & t_0 & t_2 \\ t_0 & 0 & 3 \\ t_2 & -2 & 0 \end{array}$$

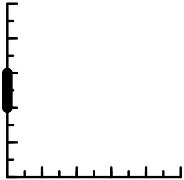


Zone After Wine Arrives



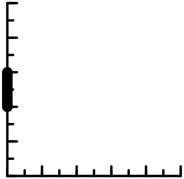
Project

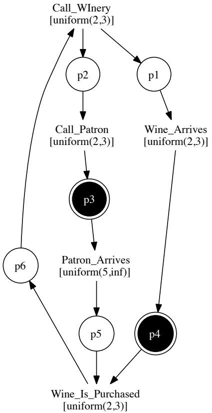
$$\begin{array}{c|cc|} & t_0 & t_2 \\ t_0 & 0 & 3 \\ t_2 & -2 & 0 \end{array}$$



Extend

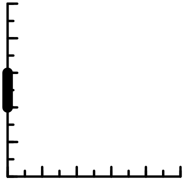
$$\begin{array}{c|ccc|} & t_0 & t_4 & t_2 \\ t_0 & 0 & 0 & 3 \\ t_4 & 0 & 0 & 3 \\ t_2 & -2 & -2 & 0 \end{array}$$





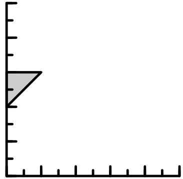
Extend

$$\begin{array}{c|ccc|} & t_0 & t_4 & t_2 \\ \hline t_0 & 0 & 0 & 3 \\ t_4 & 0 & 0 & 3 \\ t_2 & -2 & -2 & 0 \end{array}$$

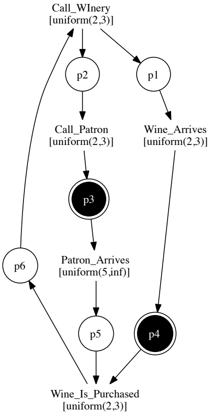


AdvTime

$$\begin{array}{c|ccc|} & t_0 & t_4 & t_2 \\ \hline t_0 & 0 & 3 & 3 \\ t_4 & 0 & 0 & 3 \\ t_2 & -2 & -2 & 0 \end{array}$$

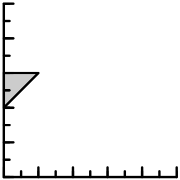


Zone After Wine Arrives



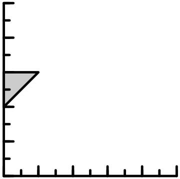
AdvTime

$$\begin{array}{c|ccc|} & t_0 & t_4 & t_2 \\ t_0 & 0 & 3 & 3 \\ t_4 & 0 & 0 & 3 \\ t_2 & -2 & -2 & 0 \end{array}$$

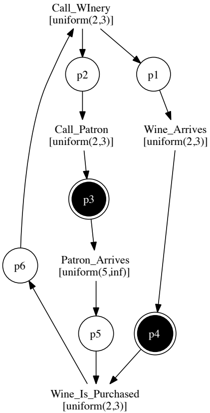


Recanon/
Norm.

$$\begin{array}{c|ccc|} & t_0 & t_4 & t_2 \\ t_0 & 0 & 1 & 3 \\ t_4 & 0 & 0 & 3 \\ t_2 & -2 & -2 & 0 \end{array}$$

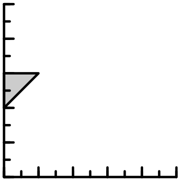


Zone After Wine Arrives and Patron is Called

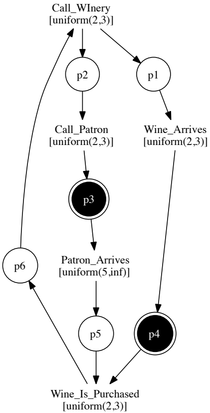


Restrict/
Reconon

$$\begin{array}{c|ccc|} & t_0 & t_4 & t_2 \\ t_0 & 0 & 1 & 3 \\ t_4 & 0 & 0 & 3 \\ t_2 & -2 & -2 & 0 \end{array}$$

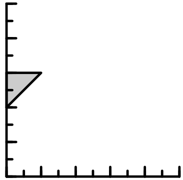


Zone After Wine Arrives and Patron is Called



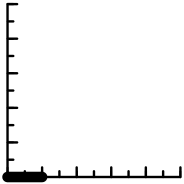
Restrict/ Reconon

$$\begin{array}{c|ccc|} & t_0 & t_4 & t_2 \\ t_0 & 0 & 1 & 3 \\ t_4 & 0 & 0 & 3 \\ t_2 & -2 & -2 & 0 \end{array}$$

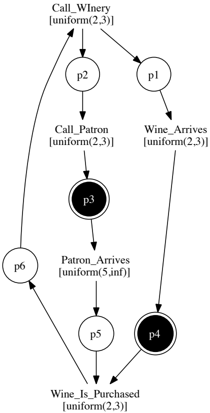


Project

$$\begin{array}{c|cc|} & t_0 & t_4 \\ t_0 & 0 & 1 \\ t_4 & 0 & 0 \end{array}$$

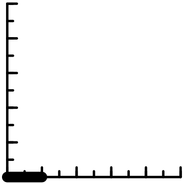


Zone After Wine Arrives and Patron is Called



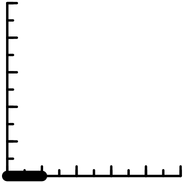
Project

$$\begin{array}{c|cc|} & t_0 & t_4 \\ t_0 & 0 & 1 \\ t_4 & 0 & 0 \end{array}$$

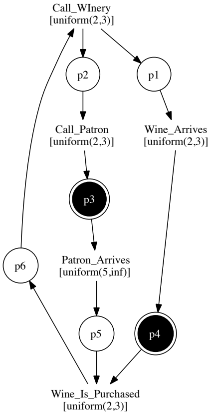


Extend

$$\begin{array}{c|ccc|} & t_0 & t_4 & t_3 \\ t_0 & 0 & 1 & 0 \\ t_4 & 0 & 0 & 0 \\ t_3 & 0 & 1 & 0 \end{array}$$

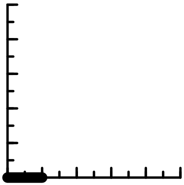


Zone After Wine Arrives and Patron is Called



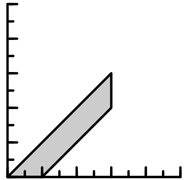
Extend

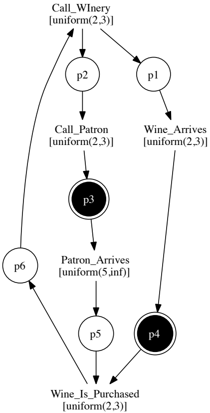
$$\begin{array}{c|ccc|} & t_0 & t_4 & t_3 \\ t_0 & 0 & 1 & 0 \\ t_4 & 0 & 0 & 0 \\ t_3 & 0 & 1 & 0 \end{array}$$



AdvTime

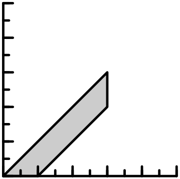
$$\begin{array}{c|ccc|} & t_0 & t_4 & t_3 \\ t_0 & 0 & 3 & \infty \\ t_4 & 0 & 0 & 0 \\ t_3 & 0 & 1 & 0 \end{array}$$





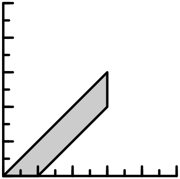
AdvTime

$$\begin{array}{c} \\ t_0 \\ t_4 \\ t_3 \end{array}\begin{array}{c} \begin{array}{ccc} t_0 & t_4 & t_3 \end{array} \\ \left|\begin{array}{ccc} 0 & 3 & \infty \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right| \end{array}$$

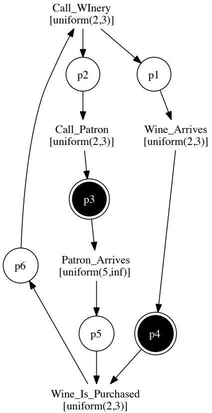


Reconon/ Norm.

$$\begin{array}{c} \\ t_0 \\ t_4 \\ t_3 \end{array}\begin{array}{c} \begin{array}{ccc} t_0 & t_4 & t_3 \end{array} \\ \left|\begin{array}{ccc} 0 & 3 & 3 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right| \end{array}$$

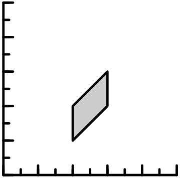


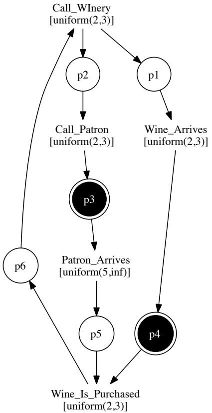
Zone After Rule Expires



Restrict

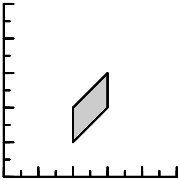
$$\begin{array}{c|ccc|} & t_0 & t_4 & t_3 \\ t_0 & 0 & 3 & 3 \\ t_4 & -2 & 0 & 0 \\ t_3 & 0 & 1 & 0 \end{array}$$





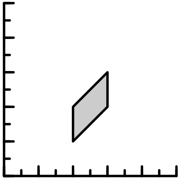
Restrict

$$\begin{array}{c|ccc|} & t_0 & t_4 & t_3 \\ t_0 & 0 & 3 & 3 \\ t_4 & -2 & 0 & 0 \\ t_3 & 0 & 1 & 0 \end{array}$$

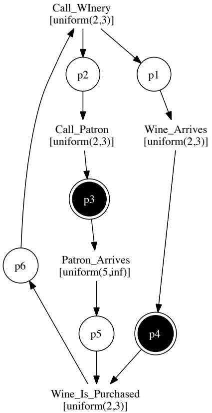


Reconon

$$\begin{array}{c|ccc|} & t_0 & t_4 & t_3 \\ t_0 & 0 & 3 & 3 \\ t_4 & -2 & 0 & 0 \\ t_3 & -1 & 1 & 0 \end{array}$$

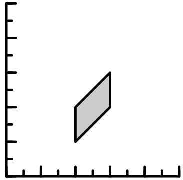


Zone After Rule Expires



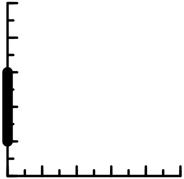
Reconon

$$\begin{array}{c|ccc|} & t_0 & t_4 & t_3 \\ t_0 & 0 & 3 & 3 \\ t_4 & -2 & 0 & 0 \\ t_3 & -1 & 1 & 0 \end{array}$$

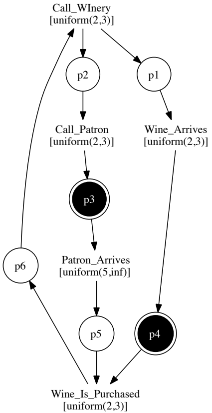


Project

$$\begin{array}{c|cc|} & t_0 & t_3 \\ t_0 & 0 & 3 \\ t_3 & -1 & 0 \end{array}$$

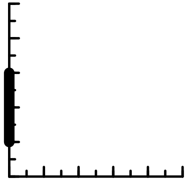


Zone After Rule Expires



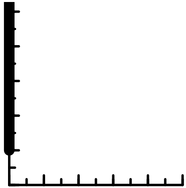
Project

$$\begin{array}{c|cc|} & t_0 & t_3 \\ t_0 & 0 & 3 \\ t_3 & -1 & 0 \end{array}$$

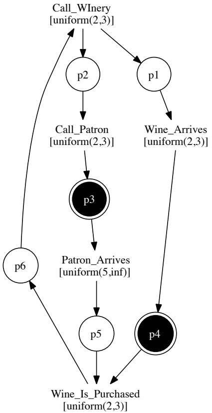


AdvTime/
Reconon

$$\begin{array}{c|cc|} & t_0 & t_3 \\ t_0 & 0 & \infty \\ t_3 & -1 & 0 \end{array}$$

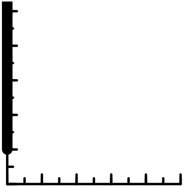


Zone After Rule Expires



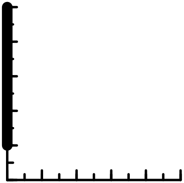
AdvTime/
Reconon

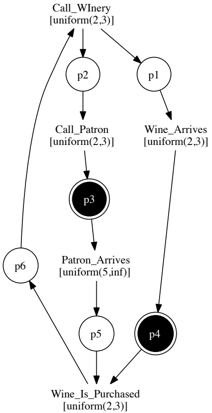
$$\begin{array}{c|cc|} & t_0 & t_3 \\ t_0 & 0 & \infty \\ t_3 & -1 & 0 \end{array}$$



Norm.

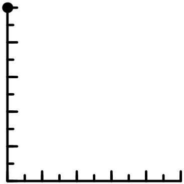
$$\begin{array}{c|cc|} & t_0 & t_3 \\ t_0 & 0 & 5 \\ t_3 & -1 & 0 \end{array}$$



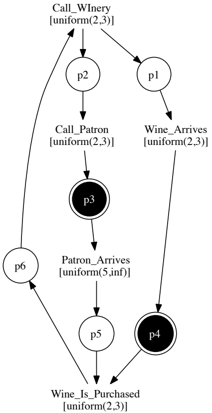


Restrict/
Recanon

$$\begin{array}{c|cc|} & t_0 & t_3 \\ t_0 & 0 & 5 \\ t_3 & -5 & 0 \end{array}$$

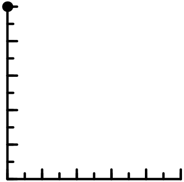


Zone After Patron Arrives



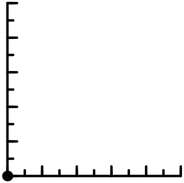
Restrict/
Recanon

$$\begin{array}{c|cc|} & t_0 & t_3 \\ \hline t_0 & 0 & 5 \\ t_3 & -5 & 0 \end{array}$$

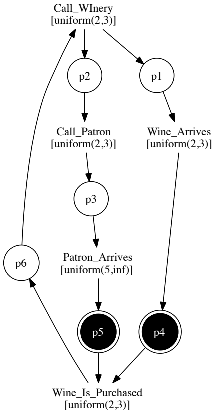


Project

$$\begin{array}{c|c|} & t_0 \\ \hline t_0 & 0 \end{array}$$

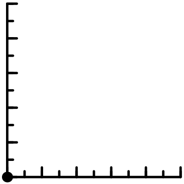


Zone After Patron Arrives



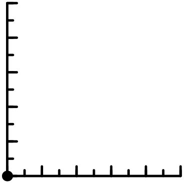
Project

$$\begin{array}{c|c|} & t_0 \\ t_0 & 0 \end{array}$$

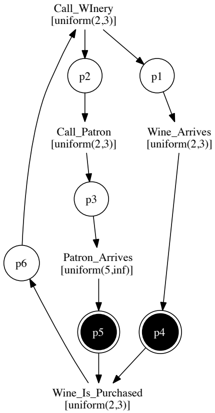


Extend

$$\begin{array}{c|cc|} & t_0 & t_5 \\ t_0 & 0 & 0 \\ t_5 & 0 & 0 \end{array}$$

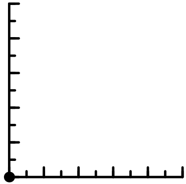


Zone After Patron Arrives



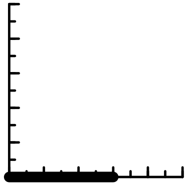
Extend

$$\begin{array}{c|cc|} & t_0 & t_5 \\ t_0 & 0 & 0 \\ t_5 & 0 & 0 \end{array}$$

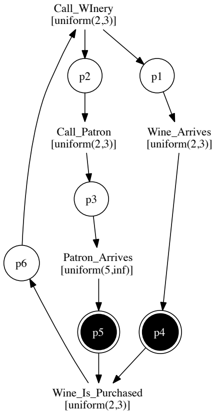


AdvTime/
Recanon/
Norm.

$$\begin{array}{c|cc|} & t_0 & t_5 \\ t_0 & 0 & 3 \\ t_5 & 0 & 0 \end{array}$$

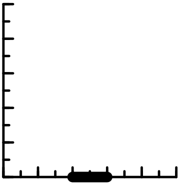


Zone After Wine is Purchased

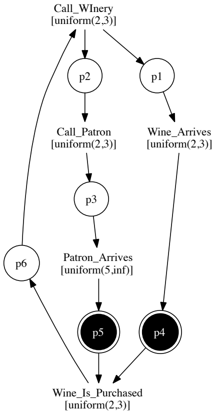


Restrict/
Recanon

$$\begin{array}{c|cc|} & t_0 & t_5 \\ \hline t_0 & 0 & 3 \\ t_5 & -2 & 0 \end{array}$$

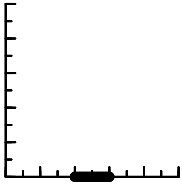


Zone After Wine is Purchased



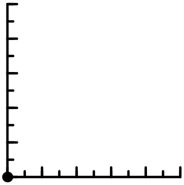
Restrict/
Recanon

$$\begin{array}{c|cc|} & t_0 & t_5 \\ \hline t_0 & 0 & 3 \\ t_5 & -2 & 0 \end{array}$$

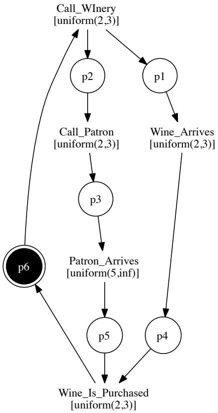


Project

$$\begin{array}{c|c|} & t_0 \\ \hline t_0 & 0 \end{array}$$

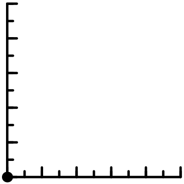


Zone After Wine is Purchased



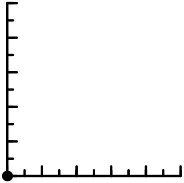
Project

$$\begin{array}{c|c|} & t_0 \\ t_0 & 0 \end{array}$$

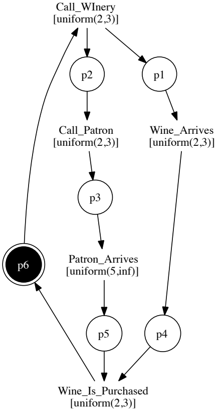


Extend

$$\begin{array}{c|cc|} & t_0 & t_6 \\ t_0 & 0 & 0 \\ t_6 & 0 & 0 \end{array}$$



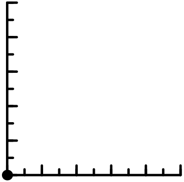
Zone After Wine is Purchased



Extend

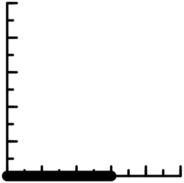
$$\begin{array}{c} \\ t_0 \\ t_6 \end{array} \begin{array}{cc} t_0 & t_6 \\ \hline \end{array}$$

$$\begin{array}{c|cc|} & t_0 & t_6 \\ t_0 & 0 & 0 \\ t_6 & 0 & 0 \end{array}$$

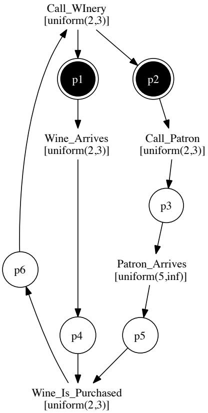


AdvTime/
Recanon/
Norm.

$$\begin{array}{c|cc|} & t_0 & t_6 \\ t_0 & 0 & 3 \\ t_6 & 0 & 0 \end{array}$$

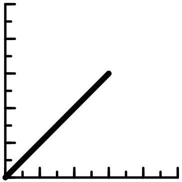


Zone After Patron is Called

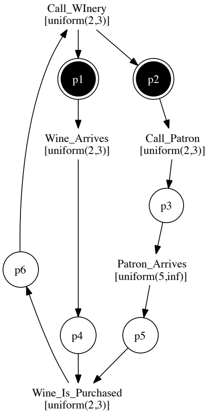


Initial

$$\begin{array}{c|ccc|} & t_0 & t_1 & t_2 \\ t_0 & 0 & 3 & 3 \\ t_1 & 0 & 0 & 0 \\ t_2 & 0 & 0 & 0 \end{array}$$

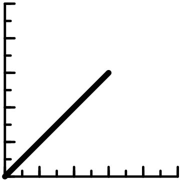


Zone After Patron is Called



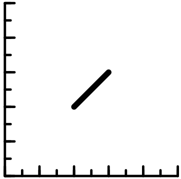
Initial

$$\begin{array}{c} \\ t_0 \\ t_1 \\ t_2 \end{array} \begin{array}{c} \begin{array}{ccc} t_0 & t_1 & t_2 \end{array} \\ \left| \begin{array}{ccc} 0 & 3 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right| \end{array}$$

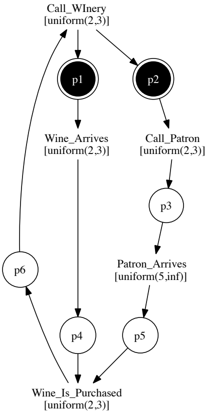


Restrict

$$\begin{array}{c} \\ t_0 \\ t_1 \\ t_2 \end{array} \begin{array}{c} \begin{array}{ccc} t_0 & t_1 & t_2 \end{array} \\ \left| \begin{array}{ccc} 0 & 3 & 3 \\ 0 & 0 & 0 \\ -2 & 0 & 0 \end{array} \right| \end{array}$$

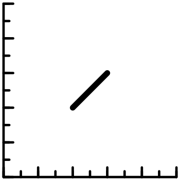


Zone After Patron is Called



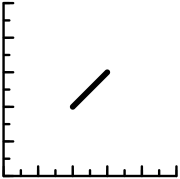
Restrict

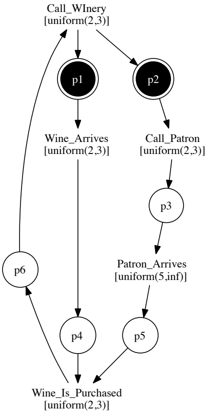
$$\begin{array}{c|ccc|} & t_0 & t_1 & t_2 \\ t_0 & 0 & 3 & 3 \\ t_1 & 0 & 0 & 0 \\ t_2 & -2 & 0 & 0 \end{array}$$



Recanon

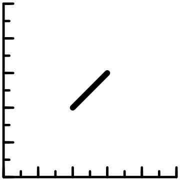
$$\begin{array}{c|ccc|} & t_0 & t_1 & t_2 \\ t_0 & 0 & 3 & 3 \\ t_1 & -2 & 0 & 0 \\ t_2 & -2 & 0 & 0 \end{array}$$





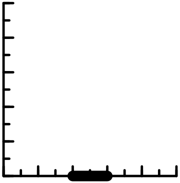
Recanon

$$\begin{array}{c|ccc|} & t_0 & t_1 & t_2 \\ t_0 & 0 & 3 & 3 \\ t_1 & -2 & 0 & 0 \\ t_2 & -2 & 0 & 0 \end{array}$$

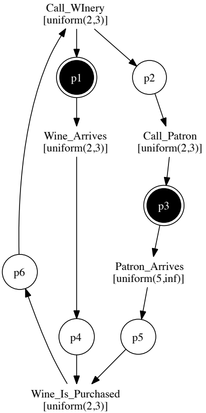


Project

$$\begin{array}{c|cc|} & t_0 & t_1 \\ t_0 & 0 & 3 \\ t_1 & -2 & 0 \end{array}$$

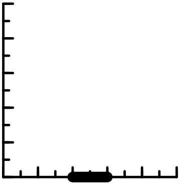


Zone After Patron is Called



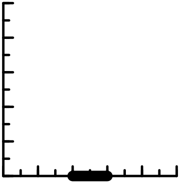
Project

$$\begin{array}{c|cc|} & t_0 & t_1 \\ t_0 & 0 & 3 \\ t_1 & -2 & 0 \end{array}$$

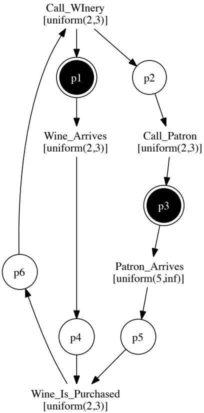


Extend

$$\begin{array}{c|ccc|} & t_0 & t_1 & t_3 \\ t_0 & 0 & 3 & 0 \\ t_1 & -2 & 0 & -2 \\ t_3 & 0 & 3 & 0 \end{array}$$

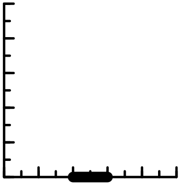


Zone After Patron is Called



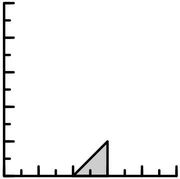
Extend

$$\begin{array}{c|ccc|} & t_0 & t_1 & t_3 \\ t_0 & 0 & 3 & 0 \\ t_1 & -2 & 0 & -2 \\ t_3 & 0 & 3 & 0 \end{array}$$

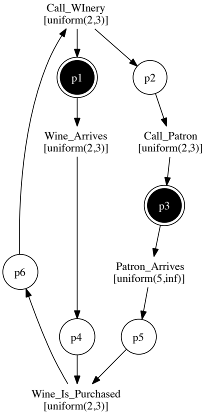


AdvTime

$$\begin{array}{c|ccc|} & t_0 & t_1 & t_3 \\ t_0 & 0 & 3 & \infty \\ t_1 & -2 & 0 & -2 \\ t_3 & 0 & 3 & 0 \end{array}$$

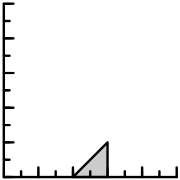


Zone After Patron is Called



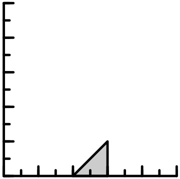
AdvTime

$$\begin{array}{c|ccc|} & t_0 & t_1 & t_3 \\ \hline t_0 & 0 & 3 & \infty \\ t_1 & -2 & 0 & -2 \\ t_3 & 0 & 3 & 0 \end{array}$$

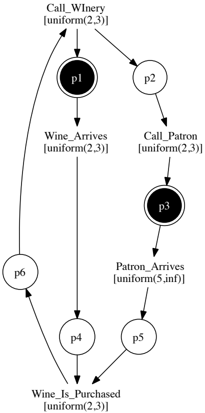


Recanon/
Norm.

$$\begin{array}{c|ccc|} & t_0 & t_1 & t_3 \\ \hline t_0 & 0 & 3 & 1 \\ t_1 & -2 & 0 & -2 \\ t_3 & 0 & 3 & 0 \end{array}$$

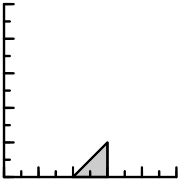


Zone After Patron is Called and Wine Arrives

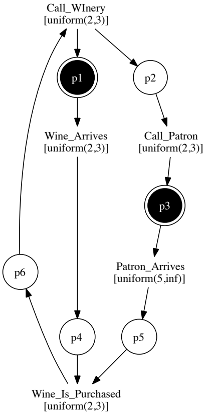


Restrict/
Recanon

$$\begin{array}{c|ccc|} & t_0 & t_1 & t_3 \\ t_0 & 0 & 3 & 1 \\ t_1 & -2 & 0 & -2 \\ t_3 & 0 & 3 & 0 \end{array}$$

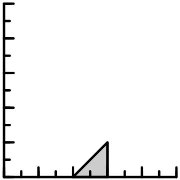


Zone After Patron is Called and Wine Arrives



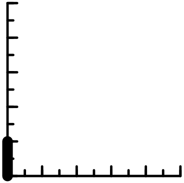
Restrict/ Recanon

$$\begin{array}{c|ccc|} & t_0 & t_1 & t_3 \\ t_0 & 0 & 3 & 1 \\ t_1 & -2 & 0 & -2 \\ t_3 & 0 & 3 & 0 \end{array}$$

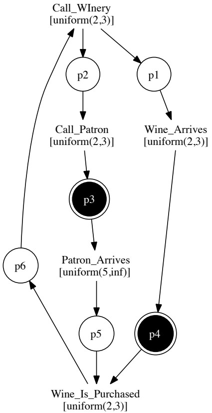


Project

$$\begin{array}{c|cc|} & t_0 & t_3 \\ t_0 & 0 & 1 \\ t_3 & 0 & 0 \end{array}$$

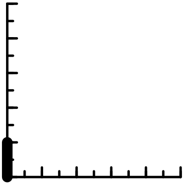


Zone After Patron is Called and Wine Arrives



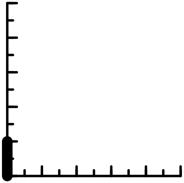
Project

$$\begin{array}{c|cc|} & t_0 & t_3 \\ t_0 & 0 & 1 \\ t_3 & 0 & 0 \end{array}$$

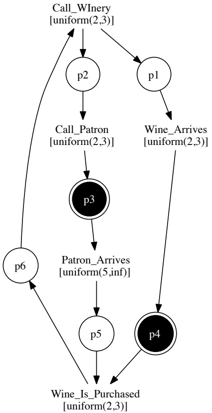


Extend

$$\begin{array}{c|ccc|} & t_0 & t_4 & t_3 \\ t_0 & 0 & 0 & 1 \\ t_4 & 0 & 0 & 1 \\ t_3 & 0 & 0 & 0 \end{array}$$

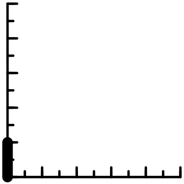


Zone After Patron is Called and Wine Arrives



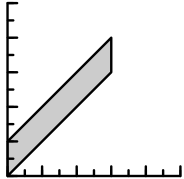
Extend

$$\begin{array}{c|ccc|} & t_0 & t_4 & t_3 \\ t_0 & 0 & 0 & 1 \\ t_4 & 0 & 0 & 1 \\ t_3 & 0 & 0 & 0 \end{array}$$

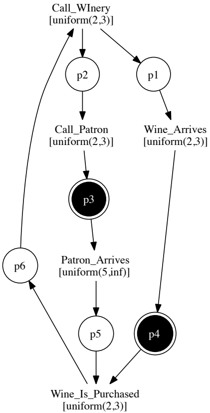


AdvTime

$$\begin{array}{c|ccc|} & t_0 & t_4 & t_3 \\ t_0 & 0 & 3 & \infty \\ t_4 & 0 & 0 & 1 \\ t_3 & 0 & 0 & 0 \end{array}$$

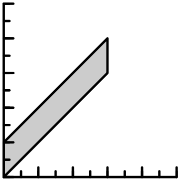


Zone After Patron is Called and Wine Arrives



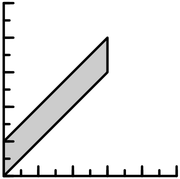
AdvTime

$$\begin{array}{c|ccc|} & t_0 & t_4 & t_3 \\ t_0 & 0 & 3 & \infty \\ t_4 & 0 & 0 & 1 \\ t_3 & 0 & 0 & 0 \end{array}$$

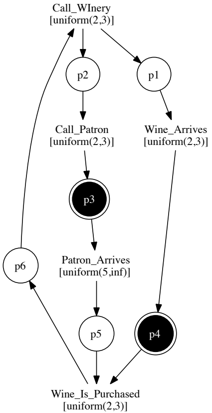


Recanon/ Norm.

$$\begin{array}{c|ccc|} & t_0 & t_4 & t_3 \\ t_0 & 0 & 3 & 4 \\ t_4 & 0 & 0 & 1 \\ t_3 & 0 & 0 & 0 \end{array}$$

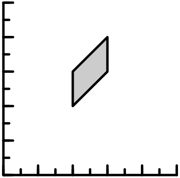


Zone After Rule Expires

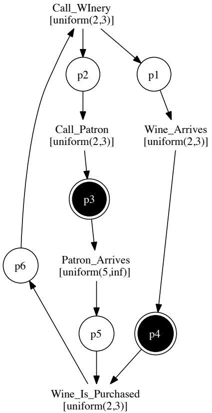


Restrict

$$\begin{array}{c|ccc|} & t_0 & t_4 & t_3 \\ t_0 & 0 & 3 & 4 \\ t_4 & -2 & 0 & 1 \\ t_3 & 0 & 0 & 0 \end{array}$$

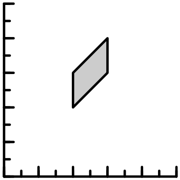


Zone After Rule Expires



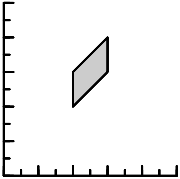
Restrict

$$\begin{array}{c|ccc|} & t_0 & t_4 & t_3 \\ t_0 & 0 & 3 & 4 \\ t_4 & -2 & 0 & 1 \\ t_3 & 0 & 0 & 0 \end{array}$$

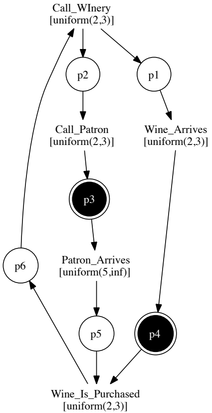


Recanon

$$\begin{array}{c|ccc|} & t_0 & t_4 & t_3 \\ t_0 & 0 & 3 & 4 \\ t_4 & -2 & 0 & 1 \\ t_3 & -2 & 0 & 0 \end{array}$$

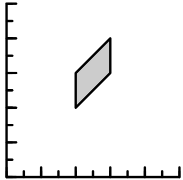


Zone After Rule Expires



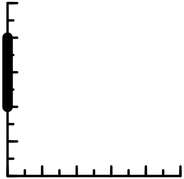
Recanon

$$\begin{array}{c|ccc|} & t_0 & t_4 & t_3 \\ t_0 & 0 & 3 & 4 \\ t_4 & -2 & 0 & 1 \\ t_3 & -2 & 0 & 0 \end{array}$$

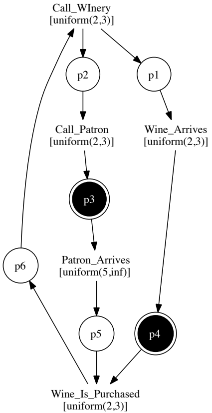


Project

$$\begin{array}{c|cc|} & t_0 & t_3 \\ t_0 & 0 & 4 \\ t_3 & -2 & 0 \end{array}$$

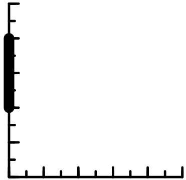


Zone After Rule Expires



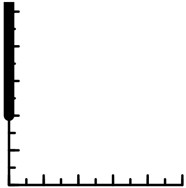
Project

$$\begin{array}{c|cc|} & t_0 & t_3 \\ \hline t_0 & 0 & 4 \\ t_3 & -2 & 0 \end{array}$$

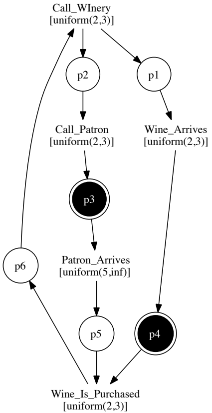


AdvTime / Recanon

$$\begin{array}{c|cc|} & t_0 & t_3 \\ \hline t_0 & 0 & \infty \\ t_3 & -2 & 0 \end{array}$$

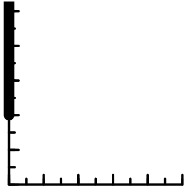


Zone After Rule Expires



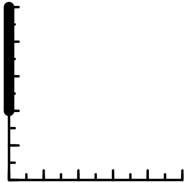
AdvTime / Recanon

$$\begin{array}{c|cc|} & t_0 & t_3 \\ t_0 & 0 & \infty \\ t_3 & -2 & 0 \end{array}$$

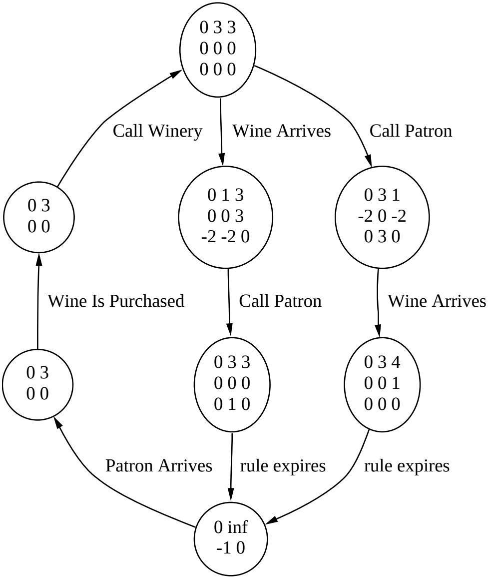


Normalize

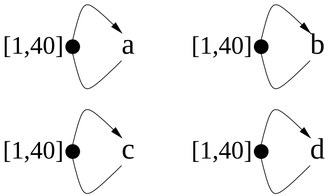
$$\begin{array}{c|cc|} & t_0 & t_3 \\ t_0 & 0 & 5 \\ t_3 & -2 & 0 \end{array}$$



Timed State Space using Zones



Adverse Example

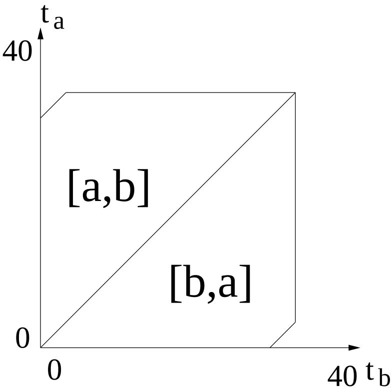


1 untimed state

2,825,761 discrete-time states

219,977,777 zones

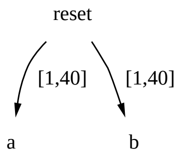
Order versus Causality



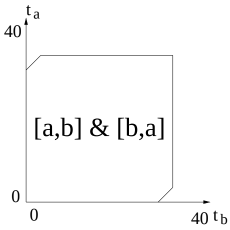
POSET Timing

- Using linear traces introduces fake orderings.
- Need to separate concurrency from casuality.
- Find zones on POSETs rather than linear traces.
- Represent POSETs using graph/matrix.

POSET Graph/Matrix/Zone

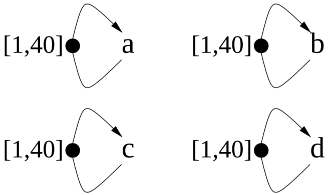


$$\begin{array}{c|ccc|} & r & a & b \\ r & 0 & 40 & 40 \\ a & -1 & 0 & 39 \\ b & -1 & 39 & 0 \end{array}$$



$$\begin{array}{c|ccc|} & t_0 & t_a & t_b \\ t_0 & 0 & 40 & 40 \\ t_a & 0 & 0 & 39 \\ t_b & 0 & 39 & 0 \end{array}$$

POSET Timing



1 untimed state
2,825,761 discrete-time states
219,977,777 zones
1 zone found using POSET timing

Creating New Zones

- If event occurs, update POSET matrix and create zone:
  - Set minimums to 0 (i.e., $m_{i0} = 0$).
  - Set maximums to the upper bound (i.e., $m_{0j} = u_j$).
  - Copy relavent time separations from POSET matrix to zone (i.e., $m_{ij} = p_{ij}$).
  - Recanonicalize.
- Otherwise, project out timer cooresponding to rule that fired.

Initial Zone using POSETs

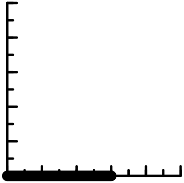
Initial POSET

$$\begin{array}{cc} & r \\ r & \left|\begin{matrix} 0 \end{matrix}\right| \end{array}$$

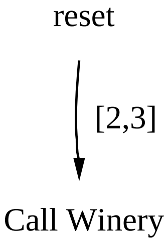
reset

Initial zone / Recanonicalize / Normalize

$$\begin{array}{cc} & \begin{matrix} t_0 & t_6 \end{matrix} \\ \begin{matrix} t_0 \\ t_6 \end{matrix} & \left|\begin{matrix} 0 & 3 \\ 0 & 0 \end{matrix}\right| \end{array}$$



Zone after the Winery is Called using POSETs



Extend POSET

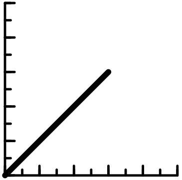
$$\begin{array}{c|cc|} & r & cw \\ \hline r & 0 & 3 \\ cw & -2 & 0 \end{array}$$

Project POSET

$$\begin{array}{c|c|} & cw \\ \hline cw & 0 \end{array}$$

Initial zone / Recanonicalize / Normalize

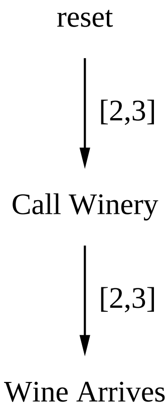
$$\begin{array}{c|ccc|} & t_0 & t_1 & t_2 \\ \hline t_0 & 0 & 3 & 3 \\ t_1 & 0 & 0 & 0 \\ t_2 & 0 & 0 & 0 \end{array}$$



POSET after the Wine Arrives

Extend POSET

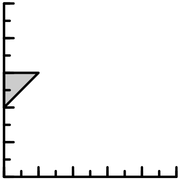
$$\begin{array}{c|cc|} & cw & wa \\ \hline cw & 0 & 3 \\ wa & -2 & 0 \end{array}$$



Zone after the Wine Arrives using POSETs

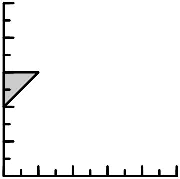
Initial zone

$$\begin{array}{c|ccc|} & t_0 & t_4 & t_2 \\ t_0 & 0 & 3 & 3 \\ t_4 & 0 & 0 & 3 \\ t_2 & 0 & -2 & 0 \end{array}$$



Recanonicalize / Normalize

$$\begin{array}{c|ccc|} & t_0 & t_4 & t_2 \\ t_0 & 0 & 1 & 3 \\ t_4 & 0 & 0 & 3 \\ t_2 & -2 & -2 & 0 \end{array}$$



POSET after the Patron is Called

Extend POSET

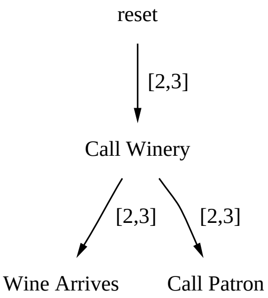
$$\begin{array}{c|ccc|} & cw & wa & cp \\ cw & 0 & 3 & 3 \\ wa & -2 & 0 & \infty \\ cp & -2 & \infty & 0 \end{array}$$

Recanonicalize

$$\begin{array}{c|ccc|} & cw & wa & cp \\ cw & 0 & 3 & 3 \\ wa & -2 & 0 & 1 \\ cp & -2 & 1 & 0 \end{array}$$

Project POSET

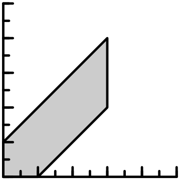
$$\begin{array}{c|cc|} & wa & cp \\ wa & 0 & 1 \\ cp & 1 & 0 \end{array}$$



Zone after the Patron is Called using POSETs

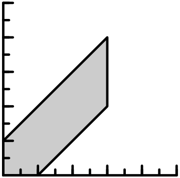
Initial zone

$$\begin{array}{c|ccc|} & t_0 & t_4 & t_3 \\ t_0 & 0 & 3 & \infty \\ t_4 & 0 & 0 & 1 \\ t_3 & 0 & 1 & 0 \end{array}$$



Recanonicalize / Normalize

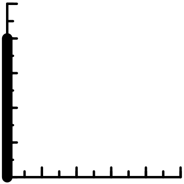
$$\begin{array}{c|ccc|} & t_0 & t_4 & t_3 \\ t_0 & 0 & 3 & 4 \\ t_4 & 0 & 0 & 1 \\ t_3 & 0 & 1 & 0 \end{array}$$



Zone after the Rule Expires using POSETs

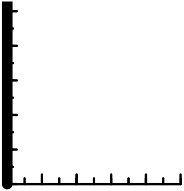
Project zone

$$\begin{array}{c|cc|} & t_0 & t_3 \\ \hline t_0 & 0 & 4 \\ t_3 & 0 & 0 \end{array}$$



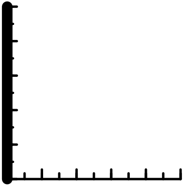
Advance time / Recanonicalize

$$\begin{array}{c|cc|} & t_0 & t_3 \\ \hline t_0 & 0 & \infty \\ t_3 & 0 & 0 \end{array}$$



Normalize

$$\begin{array}{c|cc|} & t_0 & t_3 \\ \hline t_0 & 0 & 5 \\ t_3 & 0 & 0 \end{array}$$



POSET after the Patron Arrives

Extend POSET

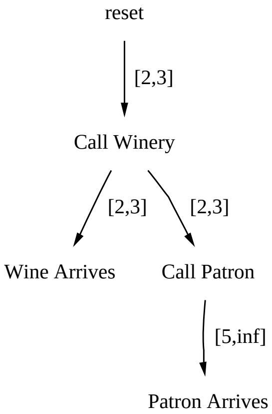
$$\begin{array}{c|ccc|} & wa & cp & pa \\ \hline wa & 0 & 1 & \infty \\ cp & 1 & 0 & \infty \\ pa & \infty & -5 & 0 \end{array}$$

Recanonicalize

$$\begin{array}{c|ccc|} & wa & cp & pa \\ \hline wa & 0 & 1 & \infty \\ cp & 1 & 0 & \infty \\ pa & -4 & -5 & 0 \end{array}$$

Project POSET

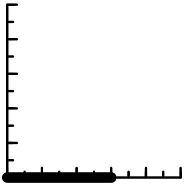
$$\begin{array}{c|c|} & pa \\ \hline pa & 0 \end{array}$$



Zone after the Patron Arrives using POSETs

Initial zone / Recanonicalize / Normalize

$$\begin{array}{c|cc|} & t_0 & t_5 \\ \hline t_0 & 0 & 3 \\ t_5 & 0 & 0 \end{array}$$



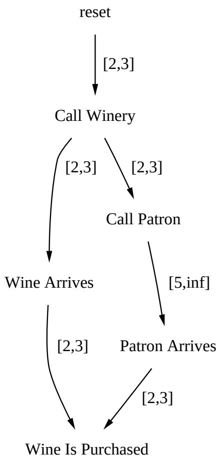
POSET after the Wine is Purchased

Extend POSET

$$\begin{array}{c|cc|} & pa & wp \\ \hline pa & 0 & 3 \\ wp & -2 & 0 \end{array}$$

Project POSET

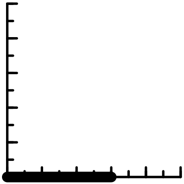
$$\begin{array}{c|c|} & wp \\ \hline wp & 0 \end{array}$$



Zone after the Wine is Purchased using POSETs

Initial zone / Recanonicalize / Normalize

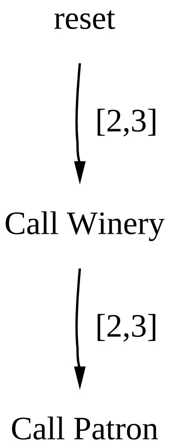
$$\begin{array}{c|cc|} & t_0 & t_6 \\ \hline t_0 & 0 & 3 \\ t_6 & 0 & 0 \end{array}$$



POSET after the Patron is Called

Extend POSET

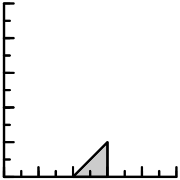
$$\begin{array}{c c} & \begin{array}{cc} cp & cw \end{array} \\ \begin{array}{c} cp \\ cw \end{array} & \left|\begin{array}{cc} 0 & -2 \\ 3 & 0 \end{array}\right| \end{array}$$



Zone after the Patron is Called using POSETs

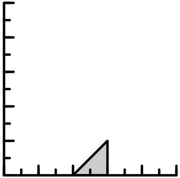
Initial zone

$$\begin{array}{c|ccc|} & t_0 & t_1 & t_3 \\ t_0 & 0 & 3 & \infty \\ t_1 & 0 & 0 & -2 \\ t_3 & 0 & 3 & 0 \end{array}$$



Recanonicalize / Normalize

$$\begin{array}{c|ccc|} & t_0 & t_1 & t_3 \\ t_0 & 0 & 3 & 1 \\ t_1 & -2 & 0 & -2 \\ t_3 & 0 & 3 & 0 \end{array}$$



POSET after the Wine Arrives

Extend POSET

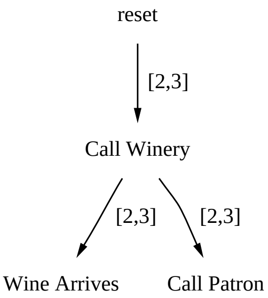
$$\begin{array}{c|ccc|} & wa & cp & cw \\ wa & 0 & \infty & -2 \\ cp & \infty & 0 & -2 \\ cw & 3 & 3 & 0 \end{array}$$

Recanonicalize

$$\begin{array}{c|ccc|} & wa & cp & cw \\ wa & 0 & 1 & -2 \\ cp & 1 & 0 & -2 \\ cw & 3 & 3 & 0 \end{array}$$

Project POSET

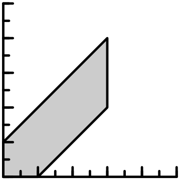
$$\begin{array}{c|cc|} & wa & cp \\ wa & 0 & 1 \\ cp & 1 & 0 \end{array}$$



Zone after the Wine Arrives using POSETs

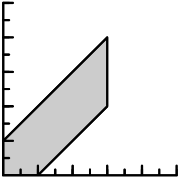
Zone

$$\begin{array}{c|ccc|} & t_0 & t_4 & t_3 \\ \hline t_0 & 0 & 3 & \infty \\ t_4 & 0 & 0 & 1 \\ t_3 & 0 & 1 & 0 \end{array}$$

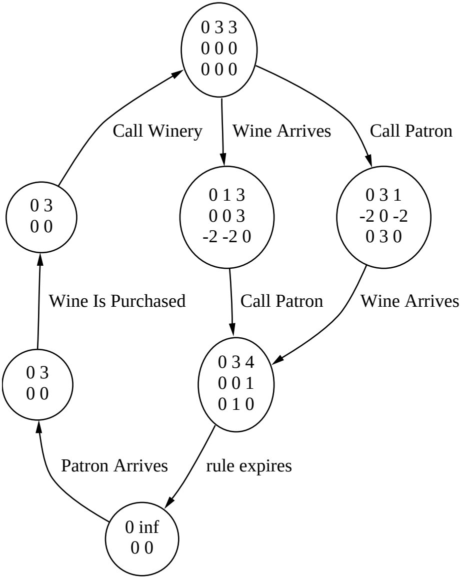


Recanonicalize /
Normalize

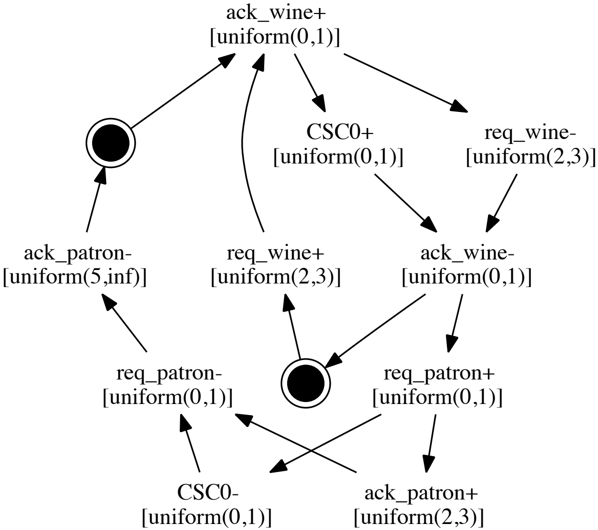
$$\begin{array}{c|ccc|} & t_0 & t_4 & t_3 \\ \hline t_0 & 0 & 3 & 4 \\ t_4 & 0 & 0 & 1 \\ t_3 & 0 & 1 & 0 \end{array}$$



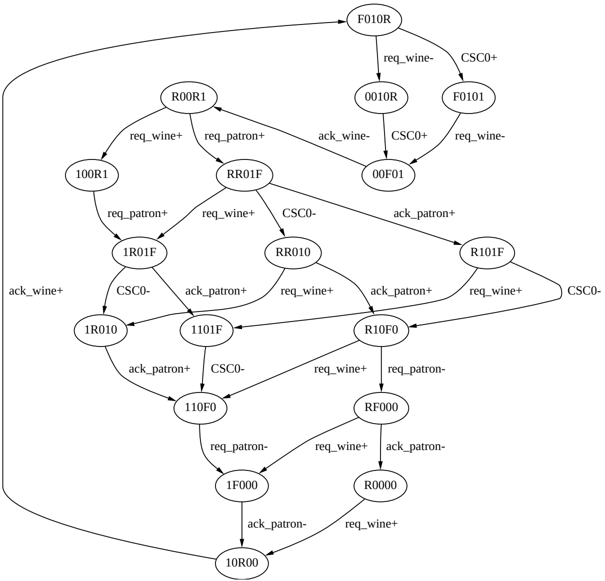
Timed State Space using POSETs



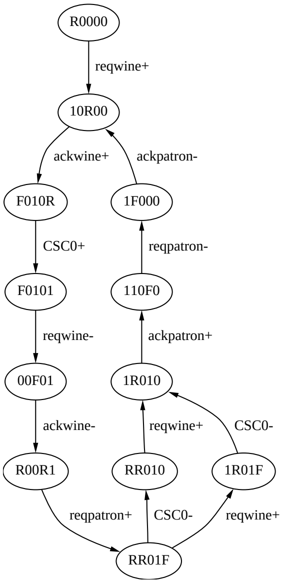
Wine Shop Example: Timed STG



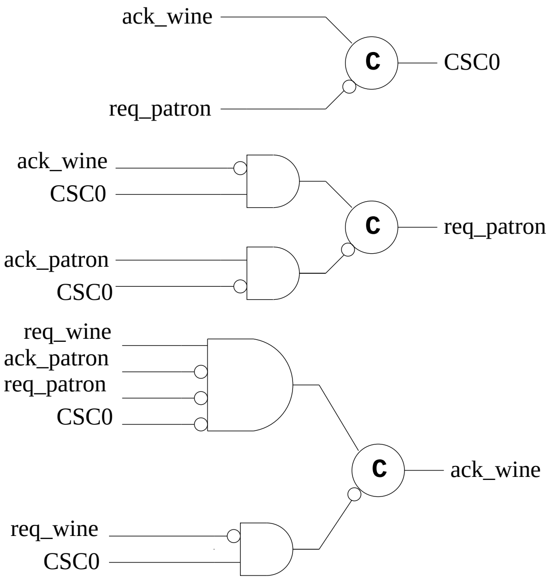
Wine Shop Example: Untimed State Graph



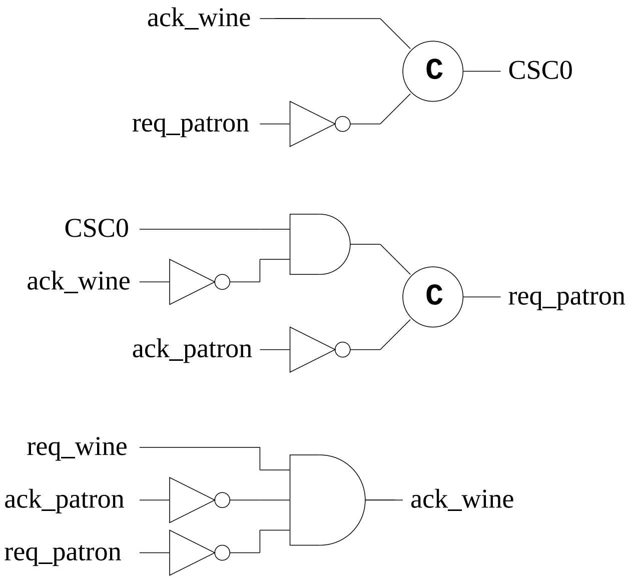
Wine Shop Example: Reduced State Graph



Wine Shop Example: Speed-Independent Circuit



Wine Shop Example: Timed Circuit



Summary

- Regions
- Discrete-time states
- Zones
- Zones + POSETs
- Timed circuits