CS 6160: Voronoi Diagrams

Due Date: .

This assignment has 6 questions, for a total of 100 points and 20 bonus points. Unless otherwise specified, complete and reasoned arguments will be expected for all answers.

Question	Points	Bonus Points	Score
Distances	20	0	
Properties	30	0	
Reconstruction	20	0	
Farthest Point Diagrams	20	0	
Minimum Enclosing ball	10	0	
More Reconstruction	0	20	
Total:	100	20	

Question 1: Distances .... **[20]**

For each of the following distance functions, describe what the corresponding Voronoi diagram will look like for n points in the plane. Your answer should describe what the 0 and 1 dimensional parts of the diagram look like, as well as give an estimate of the overall complexity of the diagram.

- $d(\mathbf{p}, \mathbf{p}) = |p_x - q_x| + |p_y - q_y|$
- $d(\mathbf{p}, \mathbf{p}) = max(|p_x - q_x|, |p_y - q_y|)$

Question 2: Properties .... **[30]**

(a) [**10**] Prove that a Voronoi cell is unbounded if and only if the site associated with that cell lies on the convex hull of the set of sites.

(b) [**10**] Prove that a Voronoi cell under the Euclidean distance must be convex.

(c) [**5**] Can you describe a distance function for which a Voronoi cell is not convex?

(d) [**5**] Argue for why computing the Voronoi diagram in the plane must take at least $\Omega(n \log n)$ time.

Question 3: Reconstruction .... **[20]**

Any convex polygon can be realized as a Voronoi cell in the Voronoi diagram of a set of points. Prove this fact by taking any convex polygon and constructing a set of points for which this polygon is one Voronoi cell.

Question 4: Farthest Point Diagrams .... **[20]**

Design an efficient algorithm to compute a farthest point Voronoi diagram. Note that only points on the convex hull will have cells in the diagram. You might consider a divide-and-conquer strategy.

Question 5: Minimum Enclosing ball .... **[10]**

Let the minimum enclosing ball of a set of points P be a ball of radius r that encloses all of P and where r is as small as possible. Assuming you're given an algorithm that can compute the furthest-point Voronoi diagram in $O(n \log n)$ time. Present an algorithm that computes the minimum enclosing ball in $O(n \log n)$ time. You can assume that the Voronoi diagram is appropriately labeled so that for each cell you know which site its points are furthest from.

BONUS Question 6: More Reconstruction .... **[20]**

Suppose you are given a Voronoi diagram in which each vertex has degree exactly three (i.e a nondegenerate case). However, you are not given the actual n sites from which the diagram was constructed. Design an algorithm that runs in linear time to reconstruct a set of sites consistent with this diagram.

HINT: Consider a single cell and a Voronoi vertex on its boundary. Find a way to determine the site corresponding to that cell.