CS 6160: Convex Hulls

Due Date: Jan 29, 2018.

This assignment has 5 questions, for a total of 100 points. Unless otherwise specified, complete and reasoned arguments will be expected for all answers.

Question	Points	Score
Hulls	20	
Who wants to sort?	20	
Merging hulls	20	
Computing Width	20	
Moment curves	20	
Total:	100	

Question 1: Hulls .... **[20]**

We are given three points in $\mathbb{R}^3$. Describe their convex hull, affine hull, conic hull, and linear hull. Remember that the x-hull is formed by taking all possible x-combinations of the points.

The handout describes the different kinds of hulls.

Solution: *insert solution here*

Question 2: Who wants to sort? .... **[20]**

Andrew's algorithm requires points to be sorted, and this is the source of the $O(n \log n)$ bound. Maybe we could do better without a sorted order. Suppose we had a simple polygon, and we ran the algorithm in the order of the points on the boundary of the polygon. Would it work ? If yes, prove it. If not, provide an example.

Question 3: Merging hulls .... **[20]**

In class we sketched out how we would merge two convex hulls into a single hull. In this exercise you will fill out the details.

(a) **[10]** Let P_1 and P_2 be two point sets in the plane that can be separated by a vertical line. Let $\mathcal{H}_1$ and $\mathcal{H}_2$ be their convex hulls. Prove that the convex hull of $P_1 \cup P_2$ consists of the edges $\mathcal{H}'_1 \cup \mathcal{H}'_2 \cup \{e_1, e_2\}$, where $\mathcal{H}'_i$ is a *connected subset of edges* of $\mathcal{H}_i$, and e_1, e_2 are edges that have one endpoint in P_1 and one endpoint in P_2 (aka *bridge* edges)

(b) **[10]** Describe *in detail, with pseudocode* how you will find this merged hull in $O(n)$ time given $\mathcal{H}_1, \mathcal{H}_2$. You should assume that the two hulls are presented as doubly-linked lists of points.

Question 4: Computing Width .... **[20]**

Let P be a set of n points in the plane. We define the *width* of P as the minimum distance between two parallel lines that enclose all of P between them.

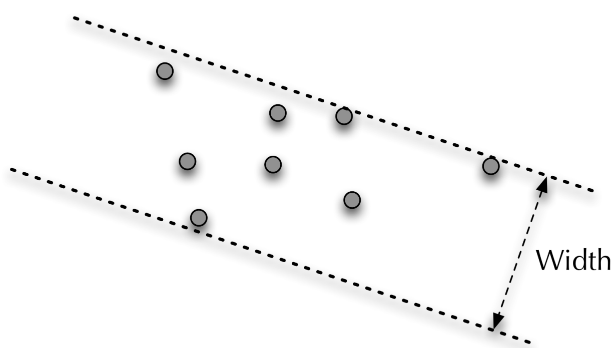

Figure 1: The width of a point set

Design an algorithm running in $O(n \log n)$ time to compute the width of a point set. **HINT:** Think about the convex hull.

Question 5: Moment curves .... **[20]**

Recall that the moment curve is defined as the function $F : \mathbb{R} \to \mathbb{R}^d$ where

$$F(t) = (t, t^2, t^3, \ldots, t^d)$$

Prove that if we take the convex hull of any n points on the moment curve, then

- Any set of $\lfloor d/2 \rfloor$ points forms a face.

- (Gale Evenness criterion): Let the set of points be $P = t_1, t_2, \ldots, t_n$. Then any d-subset T of P is a facet if and only any two elements of $P - T$ are separated by an *even* number of points from T in the sequence $t_1, \ldots, t_n$.