Lecture 5
First-Order Theories

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Last Time

- First-order theories
- Theory of equality
- Arithmetic over integers and natural numbers
  - Peano arithmetic
    - Undecidable
  - Presburger arithmetic
    - No multiplication between two variables
    - Decidable
  - Theory of integers
    - Same expressiveness as Presburger arithmetic
- Reals, rationals, arrays

This Time

▸ Homework assignment 1

▸ Planning for projects

    ▸ All final presentations on Apr 20

    ▸ Question: Can you stay later on Apr 20?

▸ Exercises with SMT solver Z3

Discussion

First-order logic

$$\forall x.\ \exists y.\ p(x, y) \rightarrow \neg p(y, x)$$

Is this formula satisfiable?

Is this formula valid?

Theory of integers

$$\forall x.\ \exists y.\ x > y \rightarrow \neg(y > x)$$

Is this formula satisfiable?

Is this formula valid?

Z3 SMT Solver

- http://rise4fun.com/z3/
- Input format is an extension of SMT-LIB standard
- Commands
  - `declare-const` – declare a constant of a given type
  - `declare-fun` – declare a function of a given type
  - `assert` – add a formula to Z3's internal stack
  - `check-sat` – determine if formulas currently on stack are satisfiable
  - `get-model` – retrieve an interpretation
  - `exit`

Linear Integer Arith. Example 1

$$x \leq y \wedge z = x + 1 \rightarrow z \leq y$$

Linear Integer Arith. Example 2

$$x \leq y \wedge z = x - 1 \rightarrow z \leq y$$

Linear Integer Arith. Example 3

$$1 \leq x \wedge x + y \leq 3 \wedge 1 \leq y \rightarrow x = 1 \vee x = 2$$

Dog, Cat, and Mouse Puzzle (from Z3 page)

- Puzzle
  - Spend exactly $100 and buy exactly 100 animals.
  - Dogs cost $15, cats cost $1, and mice cost 25 cents each.
  - You have to buy at least one of each.
  - How many of each should you buy?
- Use linear integer arithmetic
  - Hint: turn dollar amounts into cents

Scheduling Example

	Machine 1	Machine 2
Job 1	2	1
Job 2	3	1
Job 3	2	3

▸ Table gives time units required to process Job x on Machine y

▸ For a job, complete a phase on Machine 1 before starting the next on Machine 2

▸ Find using Z3 whether jobs can be scheduled in T time units

   ▸ Try T=6, T=7, T=8

Next Time

- Symbolic execution