

Lecture 8

Verification Conditions II

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Announcements

- ▶ Graded homework 1
 - ▶ Good job everyone!
 - ▶ You have **1 week** for submitting your regrading request via email
- ▶ It is time your start thinking about projects
 - ▶ Posted some ideas on canvas
 - ▶ Try to tie it to your research or interests
 - ▶ Talk to me in person
 - ▶ Next week we'll have a project brainstorming session in class

Last Time

- ▶ Simple command language
- ▶ Basic verification condition generation
- ▶ Weakest preconditions

Simple Command Language

$x := E$

havoc x

assert P

assume P

$S ; T$ [sequential composition]

$S \square T$ [choice statement]

Weakest Preconditions Cookbook

- ▶ $\text{wp}(x := E, Q) = Q[E / x]$
- ▶ $\text{wp}(\text{havoc } x, Q) = (\forall x. Q)$
- ▶ $\text{wp}(\text{assert } P, Q) = P \wedge Q$
- ▶ $\text{wp}(\text{assume } P, Q) = P \rightarrow Q$
- ▶ $\text{wp}(S ; T, Q) = \text{wp}(S, \text{wp}(T, Q))$
- ▶ $\text{wp}(S \square T, Q) = \text{wp}(S, Q) \wedge \text{wp}(T, Q)$

Checking Correctness with wp

{true}

$\text{wp}(x := 1, x + 2 = 3) = 1 + 2 = 3 \wedge \text{true}$

`x := 1;`

$\text{wp}(y := x + 2, y = 3) = x + 2 = 3 \wedge \text{true}$

`y := x + 2;`

$\text{wp}(\text{assert } y = 3, \text{true}) = y = 3 \wedge \text{true}$

`assert y = 3;`

{true}

Check: $\text{true} \rightarrow 1 + 2 = 3 \wedge \text{true}$

This Time

- ▶ If statements
- ▶ Design by contract
- ▶ Procedures

Structured if Statement

- ▶ Just a “syntactic sugar”:

if E then S else T

gets desugared into

(assume E ; S) \square (assume $\neg E$; T)

Absolute Value Example

```
if (x >= 0) {  
    abs_x := x;  
} else {  
    abs_x := -x;  
}  
assert abs_x >= 0;
```

Design by Contract

- ▶ Also called **assume-guarantee reasoning**
- ▶ Developers annotate software components with **contracts** (formal specifications)
 - ▶ Document developer's intent
 - ▶ Complex system verification broken down into compositional verification of each component
- ▶ Typical contracts
 - ▶ Annotations on procedure boundaries
 - ▶ Preconditions
 - ▶ Postconditions
 - ▶ Annotations on loop boundaries
 - ▶ Loop invariants

Design by Contract cont.

- ▶ First used in Eiffel [Bertrand Meyer]
- ▶ Native support:
 - ▶ Eiffel, Racket, SPARK Ada, Spec#, Dafny,...
- ▶ Third-party support:
 - ▶ Code Contracts project for .NET
 - ▶ Java Modeling Language
 - ▶ Contracts for Python
 - ▶ `contracts.ruby`
 - ▶ ...
- ▶ Runtime or static checking of contracts

Code Contracts Example

```
static int BinarySearch(int[] array, int value)
{
    Contract.Requires(array != null);
    ...
}
```

Spec# Example

```
static int BinarySearch(int[] a, int key)
requires forall{int i in (0: a.Length), int j in
    (i: a.Length); a[i] <= a[j]};
ensures 0 <= result ==> a[result] == key;
ensures result < 0 ==> forall{int i in (0:
    a.Length); a[i] != key};
{
    ...
}
```

Java Modeling Language (JML) Example

```
class BankingExample {  
    public static final int MAX_BAL = 1000;  
    private int balance;  
    //@ invariant balance >= 0 && balance <= MAX_BAL;  
  
    //@ ensures balance == 0;  
    public BankingExample() { this.balance = 0; }  
  
    //@ requires 0 < amount && amount+balance < MAX_BAL;  
    //@ ensures balance == \old(balance) + amount;  
    public void credit(int amount) {  
        this.balance += amount;  
    }  
}
```

Assume-Guarantee Reasoning

- ▶ Example

foo() {...}

bar() {...foo();...}

- ▶ How to verify/check bar?

Assume-Guarantee Reasoning cont.

- ▶ Solution 1
 - ▶ Inline foo
- ▶ Solution 2
 - ▶ Write contract/specification P of foo
 - ▶ Assume P when checking bar
bar() {...assume P;...}
 - ▶ Guarantee P when checking foo
foo() {...assert P;}
- ▶ Pros/cons?

Procedure

- ▶ Procedure is a complex user-defined command
 `procedure` $M(x,y,z)$ `returns` (r,s,t)
 `requires` P
 `ensures` Q
 $\{S\}$
- ▶ `requires` is a `precondition`
 - ▶ Predicate P has to hold at procedure entry
- ▶ `ensures` is a `postcondition`
 - ▶ Predicate Q has to hold at procedure exit
- ▶ S is procedure body (command)
- ▶ Note: assume procedures have no side-effects

Procedure Example

```
procedure abs(x) returns (abs_x)
requires -1000 < x && x < 1000
ensures abs_x >= 0
{
    if (x >= 0) {
        abs_x := x;
    } else {
        abs_x := -x;
    }
}
```

Desugaring Procedure Call

► **procedure** $M(x,y,z)$ **returns** (r,s,t)
requires P
ensures Q
 $\{S\}$

► **call** $a,b,c := M(E,F,G)$
desugared into:
 $x' := E; y' := F; z' := G;$
assert P' ;
assume Q' ;
 $a := r'; b := s'; c := t';$

where:

- x',y',z',r',s',t' are fresh variables
- P' is P with x',y',z' for x,y,z
- Q' is Q with x',y',z',r',s',t' for x,y,z,r,s,t

Desugaring Call Example

```
procedure abs(x) returns (abs_x)
requires -1000 < x && x < 1000
ensures abs_x >= 0
{
    if (x >= 0) {
        abs_x := x;
    } else {
        abs_x := -x;
    }
}
```

```
call a := abs(b);
assert a >= 0;
```

Desugaring Call Example

Desugaring Procedure Implementation

- ▶ `procedure M(x,y,z) returns (r,s,t)`
 `requires P`
 `ensures Q`
 `{S}`
- ▶ Implementation is correct if this is correct:
 `assume P;`
 `S;`
 `assert Q;`

Desugaring Implementation Example

```
procedure abs(x) returns (abs_x)
requires -1000 < x && x < 1000
ensures abs_x >= 0
{
    if (x >= 0) {
        abs_x := x;
    } else {
        abs_x := -x;
    }
}
```

Desugaring Implementation Example