Lecture 7

Verification Conditions I

Zvonimir Rakamarić
University of Utah

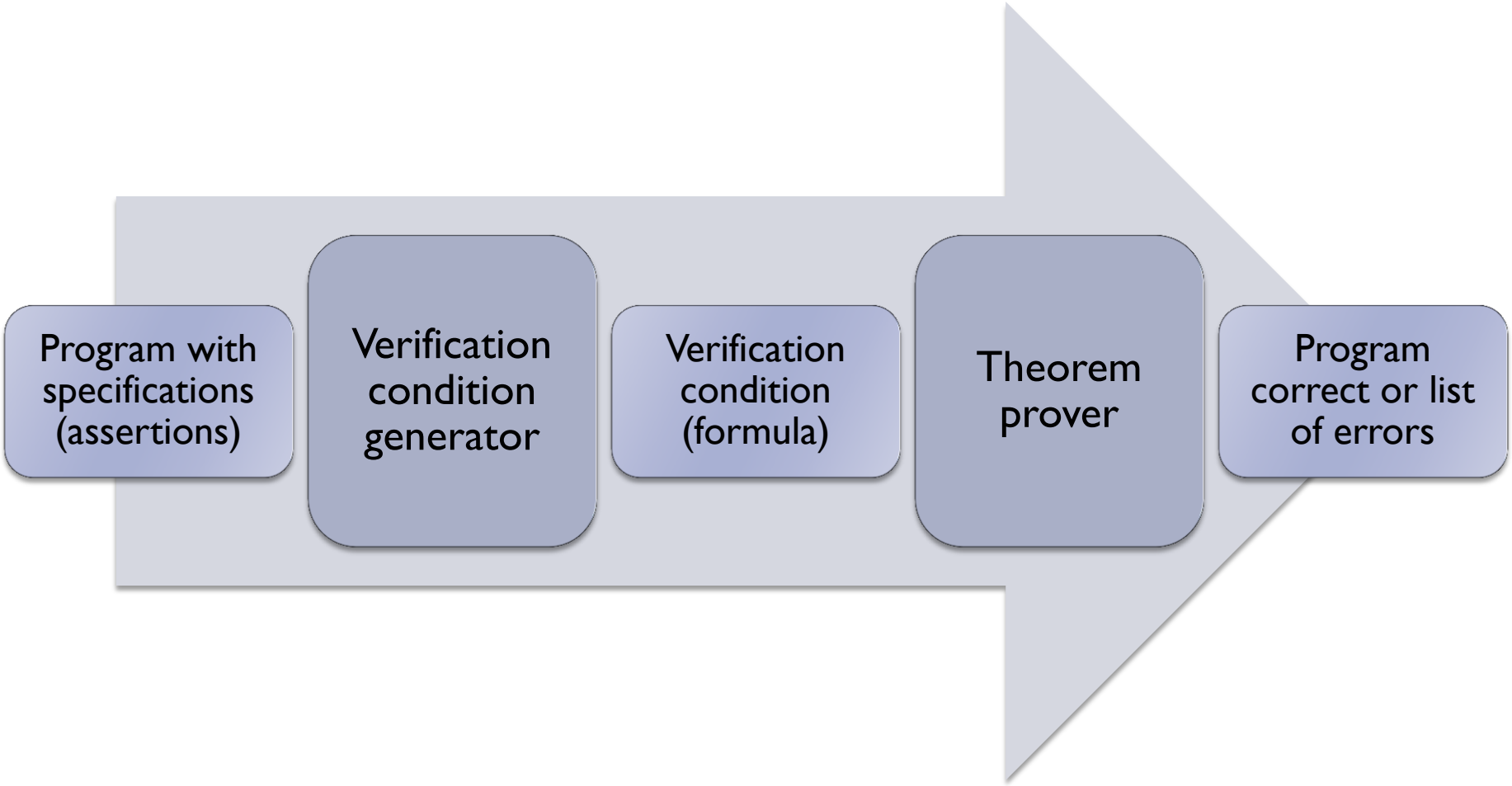
Last Time

- First-order logic
- First-order theories
- Using Z3

This Time

- Verification condition generation
- Weakest precondition transformer
- Section 5 in our textbook
- Homework 2 is posted

Basic Verifier Architecture



Verification Condition Generator

- Creates verification conditions (mathematical logic formulas) from program's source code
  - If VC is valid – program is correct
  - If VC is invalid – possible error in program
- Based on the theory of Hoare triples
  - Formalization of software semantics for verification
- Verification conditions computed automatically using *weakest preconditions* (wp)

Simple Command Language

x := E

havoc x

assert P

assume P

S ; T [sequential composition]

S □ T [choice statement]

Program States

- Program state *s*
  - Assignment of values (of proper type) to all program variables
  - Sometimes includes program counter variable *pc*
    - Holds current program location
- Example

  $s : (x \mapsto -1, y \mapsto 1)$

  $s : (pc \mapsto \mathrm{L}, a \mapsto 0, i \mapsto 3)$
- Reachable state is a state that can be reached during some computation

Program States cont.

- A set of program states can be described using a FOL formula
- Example

  Set of states:

  s : { $(x \mapsto 1)$, $(x \mapsto 2)$, $(x \mapsto 3)$ }

  FOL formulas defining s:

  $x = 1 \vee x = 2 \vee x = 3$

  $0 < x \wedge x < 4$ [if x is integer]

Hoare Triple

- Used for reasoning about (program) executions

$$\{ P \}\ S\ \{ Q \}$$

- S is a command
- P is a precondition – formula about program state before S executes
- Q is a postcondition – formula about program state after S executes

Hoare Triple Definition

$$\{ P \}\ S\ \{ Q \}$$

- When a state *s* satisfies precondition P, every terminating execution of command S starting in *s*
  - does not go wrong, and
  - establishes postcondition Q

Hoare Triple Examples

- {a = 2} b := a + 3; {b > 0}
- {a = 2} b := a + 3; {b = 5}
- {a > 3} b := a + 3; {a > 0}
- {a = 2} b := a * a; {b > 0}

Weakest Precondition [Dijkstra]

- The most general (i.e., weakest) P that satisfies

$$\{ P \}\ S\ \{ Q \}$$

  is called the weakest precondition of S with respect to Q, written:

$$wp(S, Q)$$

- To check $\{ P \}\ S\ \{ Q \}$ prove $P \rightarrow wp(S, Q)$
- Example

$$\{?P?\}\ \ b := a + 3; \{b > 0\}$$

$$\{a + 3 > 0\}\ \ b := a + 3; \{b > 0\}$$

$$wp(b := a + 3, b > 0) = a + 3 > 0$$

Strongest Postcondition

- The strongest Q that satisfies

  $$\{ P \} S \{ Q \}$$

  is called the strongest postcondition of S with respect to P, written:

  $$sp(S, P)$$

- To check $\{ P \} S \{ Q \}$ prove $sp(S, P) \rightarrow Q$
- Strongest postcondition is (almost) a dual of weakest precondition

Weakest Preconditions Cookbook

- wp(x := E, Q) = $Q[E / x]$
- wp(havoc x, Q) = $(\forall x . Q)$
- wp(assert P, Q) = $P \wedge Q$
- wp(assume P, Q) = $P \rightarrow Q$
- wp(S ; T, Q) = wp(S, wp(T, Q))
- wp(S □ T, Q) = $wp(S, Q) \wedge wp(T, Q)$

Checking Correctness with wp

{true}

```
x := 1;

y := x + 2;

assert y = 3;
```

{true}

Checking Correctness with wp cont.

{true}

wp(x := 1, x + 2 = 3) = 1 + 2 = 3 $\wedge$ true

```
x := 1;
```

wp(y := x + 2, y = 3) = x + 2 = 3 $\wedge$ true

```
y := x + 2;
```

wp(assert y = 3, true) = y = 3 $\wedge$ true

```
assert y = 3;
```

{true}

Check: true $\rightarrow$ 1 + 2 = 3 $\wedge$ true

Example II

{x > 1}

```
y := x + 2;

assert y > 3;
```

{true}

Example II cont.

{x > 1}

wp(y := x + 2, y > 3) = x + 2 > 3

```
y := x + 2;
```

wp(assert y > 3, true) = y > 3 ∧ true = y > 3

```
assert y > 3;
```

{true}

Check: x > 1 → (x + 2 > 3)

Example III

{true}

```
assume x > 1;

y := x * 2;

z := x + 2;

assert y > z;
```

{true}

Example III cont.

{true}

wp(assume x > 1, x * 2 > x + 2) = x>1 $\rightarrow$ x*2 > x+2

```
assume x > 1;
```

wp(y := x * 2, y > x + 2) = x * 2 > x + 2

```
y := x * 2;
```

wp(z := x + 2, y > z) = y > x + 2

```
z := x + 2;
```

wp(assert y > z, true) = y > z $\wedge$ true = y > z

```
assert y > z;
```

{true}

Structured if Statement

- Just a “syntactic sugar”:

  if E then S else T

  gets desugared into

  (assume E ; S) □ (assume ¬E ; T)

Absolute Value Example

```
if (x >= 0) {
   abs_x := x;
} else {
   abs_x := -x;
}
assert abs_x >= 0;
```

Next Time

- Procedures
- Loops
- Loop invariants