$$T(n) = (\sqrt{n})^{\sqrt{T(n-1)}}$$

$$T^2(n) = n^{\sqrt{T(n-1)}}$$

$$g(n) = \log T(n)$$

$$2^{g(n)} = (\sqrt{n})^{\left(\sqrt{2^{g(n-1)}}\right)}$$

AF

$$\log T(n) = \frac{1}{2}\sqrt{T(n-1)}\log n$$

$$g(n) = \frac{1}{2}\sqrt{2^{g(n-1)}}\log n$$

$$\log T(n) = \sqrt{T(n-1)}$$

$$\log^2 T(n) = T(n-1)$$

$$\log T(n) = T(n-1)$$

$$T(n) = 2^{T(n-1)}$$

$$\log^2 T(n) = T(n-1) \qquad \log T(n) \cdot \log T(n) \cdot$$

$$T(1) = \left. 2^{2^{\cdot^{\cdot^{\cdot}}}} \right\} n \qquad \boxed{T(n) = 2^{\sqrt{T(n-1)}}}$$

$$T(n) = \left. 2^{2^{2}} \right\} n-1 \qquad T(n) = 2^{\sqrt{T(n-1)}}$$

$$\left. 2^{2^{2}} \right\} n-1 \qquad T(n) = 2^{\sqrt{2^{\sqrt{T(n-2)}}}}$$

$$T(n) = \left. 2^{\sqrt{2^{\sqrt{2^{\sqrt{}}}}}} \right\} n \text{ times}$$

$$g(n) = \cancel{\Delta}\sqrt{2^{g(n-1)}} \quad \underline{\underline{\log n}}$$

$$e^{\sqrt{e^{\int}}} \quad \}_n$$

$$g(n) = \sqrt{2^{g(n-1)} \lg^2 n} \rightarrow \qquad \lg^2 n = 2^{(2 \lg\lg n)}$$

$$= \sqrt{2^{g(n-1) \cdot 2 \cdot \lg\lg n}}$$

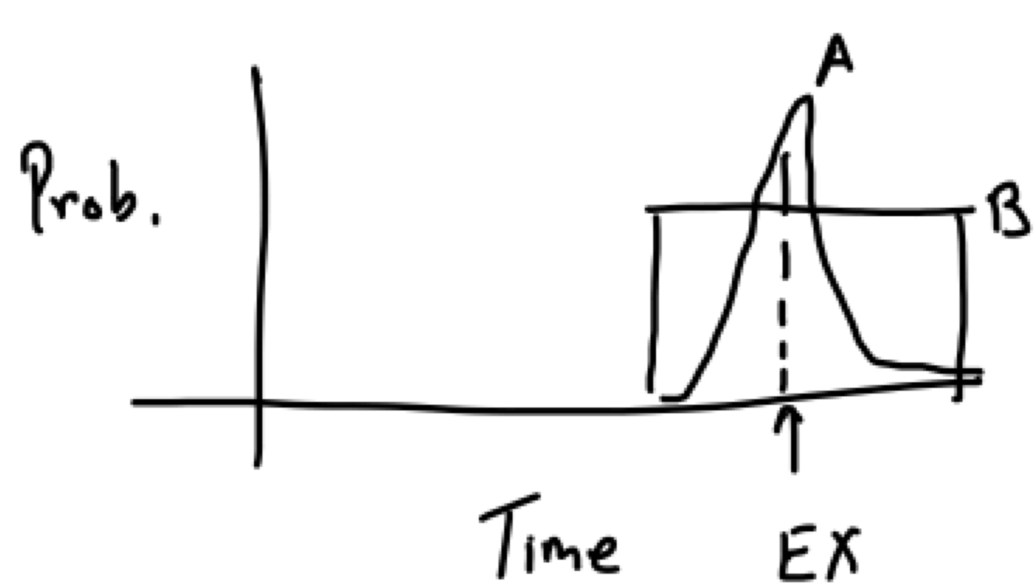
$$T(n) \leq 2^{\sqrt{1.1 T(n-1)}}$$

$$T(n) = \sqrt{2^{T(n-1)}}$$

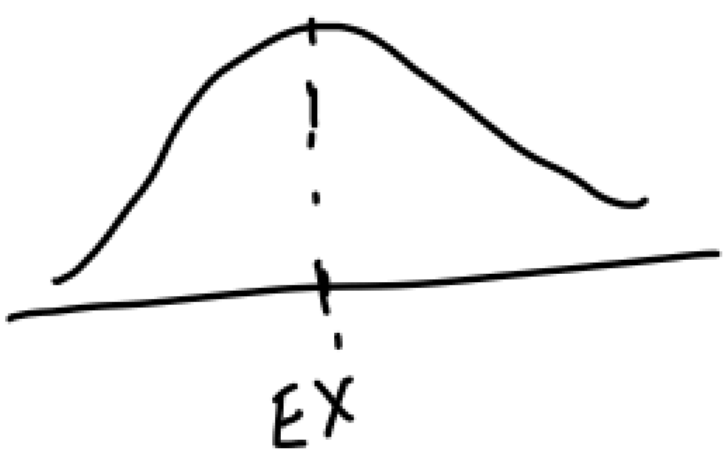
$$T(n) = \sqrt{2^{T(n-1) + \lg\lg n}}$$

Probabilistic Behavior

- Resource usage
- Correctness



Random variable



$Pr(X >> \mu)$?

Markov Inequality

Let X be a positive r.v., $EX = \mu$

Then $\forall a > 0$

$$\Pr(X \geq a\mu) \leq \frac{1}{a}$$

Sup. $\Pr(X \geq a\mu) > \frac{1}{a}$

$$EX. \quad \underbrace{\qquad\qquad}_{X < a\mu} + \overbrace{\underbrace{\qquad\qquad}_{X \geq a\mu}}^{> \mu} > \mu$$

n coins: all fair

$Pr(\text{seeing} \geq \frac{3n}{4} \text{ heads})$

$Pr(X \geq \frac{3n}{4}) \leq \frac{2}{3}$

$X_i = 1$ if i^{th} toss = heads

$\#\text{heads} = \sum X_i = X$

$EX = \sum E X_i = \frac{n}{2}$

$\mu = \frac{n}{2}$, $a\mu = \frac{3n}{4}$

$a = \frac{3}{2}$

Chebyshev Inequality

$X_1 + X_2$ independent

if $Pr(X_1 = x_1, X_2 = x_2)$

$= Pr(X_1 = x_1)\, Pr(X_2 = x_2)$

$Var(X) = EX^2 - (EX)^2 = \sigma^2$

$\sigma = \sqrt{Var(X)}$

If $X_1 \dots X_n$ are independent

$$Var(\Sigma X_i) = \Sigma Var(X_i)$$

$\rightarrow \forall a > 0,$

$$Pr(|X - \mu| \geq a\sigma) \leq \frac{1}{a^2}$$

$$Y = (X - EX)^2$$

$$X = \Sigma X_i$$

$$Var(X) = Var(\Sigma X_i)$$

$$= \Sigma Var(X_i)$$

X_i are independent

$$Var(X_i) = EX_i^2 - (EX)^2$$

$$= 1/2 - 1/4$$

$$= 1/4$$

$$Var(x) = \frac{n}{4} = \sigma^2$$

$$Pr(|x - \mu| \geq a) \leq \frac{\sigma^2}{a^2}$$

$\frac{n}{2}$ $\frac{n}{4}$

$$\leq \frac{(n/4)}{(n/4)^2} = \frac{4}{n}$$

$$M.B \leq \frac{2}{3}$$

$$C.B \leq \frac{4}{n}.$$

Chernoff bound

Hoeffding bound

Bernstein's inequality

McDiarmid's inequality

⋮

Let $X_1, \ldots X_n$ be 0,1

r.v.s $\quad Pr(X_i = 1) = p_i$

$EX_i = p_i$

$X = \sum X_i \qquad \mu = EX = \sum p_i$

$$Pr\left(X > (1+\delta)\mu \right) \le c_\delta^{\mu}$$

$$c_\delta = \left[\frac{e^\delta}{(1+\delta)^{1+\delta}} \right]$$

$$\Pr(|X-\mu| \geq \delta\mu) \leq 2e^{-\frac{\mu\delta^2}{3}}$$

$$\Pr\left(|X-\mu| \geq \frac{n}{4}\right) \leq 2 \cdot e^{-\left[\frac{n}{2}\cdot\frac{1}{3}\cdot\frac{1}{4}\right]}$$

$\mu = n/2, \quad \delta = 1/2$ $\qquad = 2e^{-n/24}$

MB - $\leq \frac{2}{3}$

Cheb $\leq \frac{4}{n}$

Cher. $\leq 2 \cdot e^{-n/24}$

$\Pr(|X-\mu| \geq \sqrt{n \ln n})$

$\leq \frac{2}{n}$

Throw n balls in n bins:

$$E[\ell] = 1$$

$$\max \ell \approx \frac{\log n}{\log\log n}$$

Throws $n \log n$ balls in n bins?

$$E[\ell] = \log n$$

$$\text{Prob.}[\ell > 1.5 \log n] \ll \frac{1}{n}$$

$$\text{Prob}[\ell < 0.5 \log n] \ll \frac{1}{n}$$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