Electronics for Computer Scientists

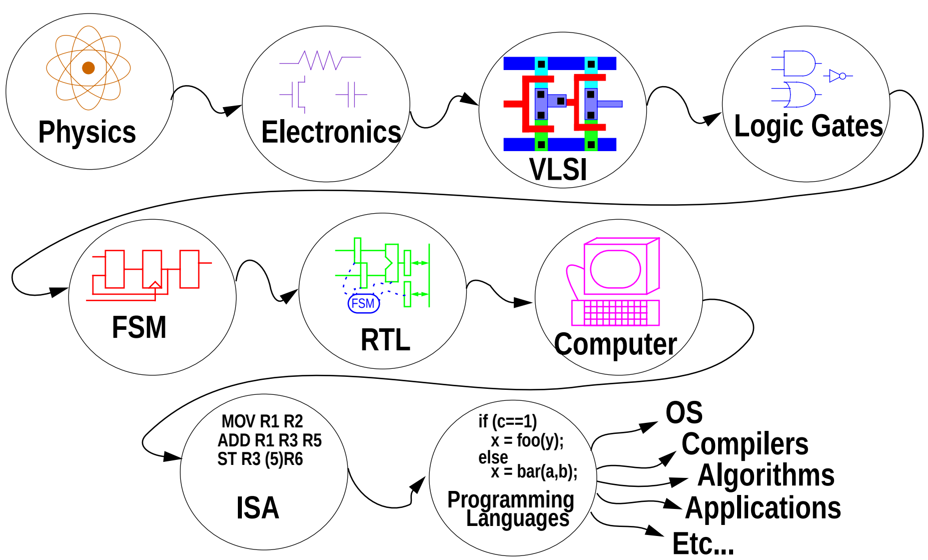
Ohm's Law to VLSI

Erik Brunvand

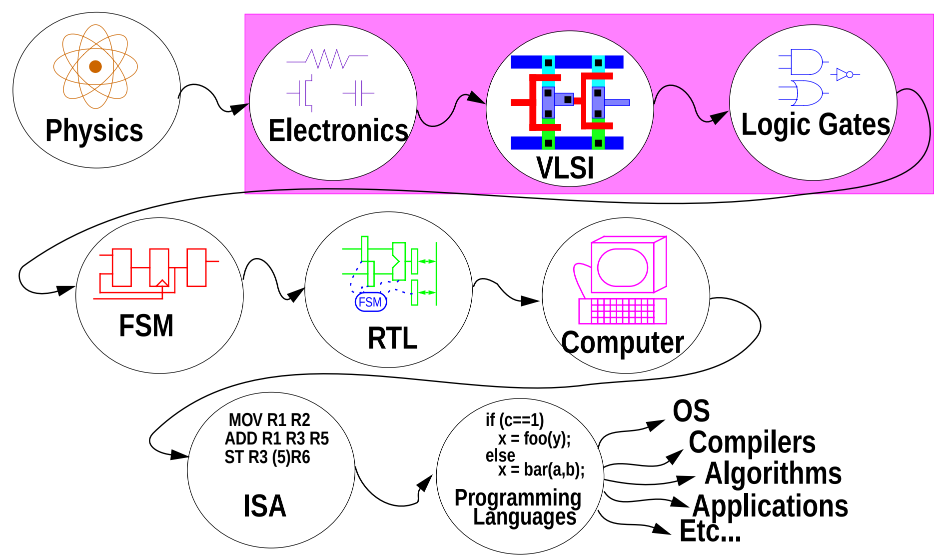
Why do we want to know?

- It's important to know something about computing hardware
- If only to not sound like a dummy...
  - How much power does your PC draw?
  - Why does your laptop only last a hour on a battery, but your watch lasts 2 years?
  - Why does a faster processor burn more power?
  - 700MHz is pretty fast. What are the issues in making things go faster?
  - How are logic gates built? How do they work?
  - How are logic gates used to build computing systems?
- It also lets you understand and appreciate limitations and advances in hardware

The Big Picture



This Talk



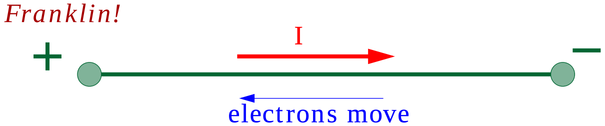
Electric Charge

- ❑ Atomic-level property
  - Positive charge = Proton
  - Negative charge = Electron
- ❑ Charges produce force against each other
  - Like charges repel
  - Different charges attract
- ❑ SI unit of charge is Coulomb (Q, q are quantity symbols)
  - Charge on electron is -1.602×10^{-19} Coulombs
  - 6.241×10^{18} electrons = 1 Coulomb

Electric Current

Results from charge moving in a conductor

- ❑ SI unit of current is Ampere, Amp, A (I, i are quantity symbols)
  - 1 Amp is 1 Coulomb of charge passing a point in 1 second
  - I (Amperes) = Q (Coulombs) / t (seconds)
- ❑ Current has a direction: it flows from positive to negative points (positive current)
  - But, electrons are really the things that move in the conductor
  - And, they move from negative to positive
  - So, the electrons move in the opposite direction as current flow
  - *Blame Ben Franklin!*



Voltage

Difference in electrical potential at two points in a circuit

- ❑ A measure of how much work is involved in moving charge between those points
  - W (joules) = F (newtons) * s (meters)
- ❑ Energy is the capacity to do work.
  - Potential energy is energy something has because of position
  - Voltage difference is a potential difference
- ❑ Voltage is the energy that causes current to flow
  - Current flows from higher potential to lower potential

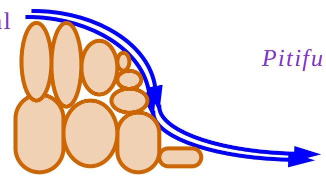
Voltage is Relative

Measured relative to two points in a system

- ❑ 1 Volt is the work required to move 1 Coulomb of charge from one point to another
  - $V_{a\text{-}b}$ (volts) = W (joules) / Q (Coulombs)
- ❑ Raising the voltage of one Coulomb of charge by 1 volt takes 1 joule of energy...
- ❑ One point is arbitrarily called 0v or Ground (GND)
  - Which means that voltage can easily be negative with respect to that arbitrary point

Water Analogy

- Current flow = water flow
- Amount of current = how much water
- Voltage = potential energy of the water
  - 0v = stagnant pool of water, no flow
  - Small voltage = tiny waterfall, not much energy
  - Large voltage = large waterfall, lots of energy
  - Negative voltage = dig a hole under the pond
- More water analogy later....



Power

The rate at which something produces or consumes energy

- P (watts) = W (joules) / t (seconds)

$$P\text{ (watts)} = \frac{W\text{ (joules)}}{Q\text{ (coulombs)}} * \frac{Q\text{ (coulombs)}}{t\text{ (seconds)}}$$

P (watts) = V (volts) * I (Amperes)

Example

- How much current flows in a light bulb from a steady movement of 10^{22} electrons in 1 hour?

$$\frac{10^{22}\text{ electrons}}{1\text{h}} * \frac{1\text{h}}{3600\text{s}} * \frac{-1.602\text{x}10^{-19}\text{ C}}{1\text{ electron}} = -0.445\text{C/s}$$

$$= -0.445\text{A}$$

Example

- How much current does a 1200w toaster draw from a 120v power connection?

P = V I

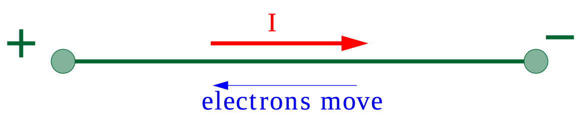
I = P/V = 1200w/120v = 10A

How fast do electrons move?

What is the "drift velocity" of an electron?

- ❑ Example: 14 gauge copper wire, 10A current
  - Copper wire has 1.38×10^{24} free electrons/in^3
  - 14 gauge cross section is $3.23/10^{-3}$ in^2
  - Electron velocity is (current)/(area * electron density)

Electrical impulse moves at 2.998×10^8 m/s
(i.e. close to speed of light)



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$$\text{velocity} = \frac{10C}{1s} * \frac{1}{3.23 \times 10^{-3} in^2} * \frac{1 in^3}{1.38 \times 10^{24} \text{ electrons}}$$

$$= \frac{10C}{1s} * \frac{1}{3.23 \times 10^{-3} in^2} * \frac{1 in^3}{1.38 \times 10^{24} \text{ electrons}} * \frac{0.0254m}{1in} * \frac{1 \text{ electron}}{-1.602 \times 10^{-19} C}$$

= -3.56x10-4 m/s * 3600s/h = -1.28m/h (Very slow!!!)

Resistance

The property that opposes or resists current flow

- ❑ Water analogy:
  - friction of water in a small pipe
- ❑ Electronics:
  - Electrons collide with conductor atoms and lose energy in the form of heat
- ❑ Current is proportional to applied voltage
  - Unit is the Ohm, symbol is Ω
  - Ohm's Law: I (amps) = V (volts) / R (Ohms)
  - I = V/R or V = I R

Resistance of Materials

Proportional to length
inversely proportional to cross-section area

- ❑ Big Pipe = less force (voltage) required to push water (current) through
- ❑ Little Pipe = more force (voltage) required to force the same amount of current through
  - Resistance = ρ (L / A) where ρ is "resistivity"in Ωm

Material	Resistivity	Material	Resistivity
Silver	1.64×10^{-8}	Nichrome	100×10^{-8}
Copper	1.72×10^{-8}	Silicon	2500
Aluminum	2.83×10^{-8}	Quartz	10^{17}

(note, this property is measurable over 25 orders of magnitude!)

Example

- ❑ Given a 240v heating element in a stove that has 24 Ω resistance, what fuse to use?
  - Fuse must be able to carry the current of the heating element
  - $I = V / R = 240v / 24\Omega = 10A$
- ❑ How much power does this heating element dissipate?
  - Recall $P = V I$, and $V = I R$, so $P = I^2 R$
  - So $P = 10^2 * 24W = 2400$ W

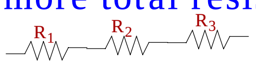
Example

- ❑ What is the resistance of an Al wire 1000m long with diameter 1.626mm?
  - Cross sectional area = Πr^2, $r=d/2 = 0.813\times10^{-3}$m
  - R (ohms) = $\rho (L / A)$

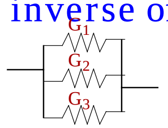
$$= \frac{(2.83\times10^{-8}\Omega m)\ (1000m)}{\Pi\ (0.813\times10^{-3}m)^2} = 13.6\Omega$$

Series and Parallel Connections of Resistors

- ❑ Resistors in series = more total resistance
  - $R_{tot} = R_1 + R_2 + ... + R_n$



- ❑ Resistors in parallel = less total resistance
- ❑ Think about conductance as the inverse of resistance



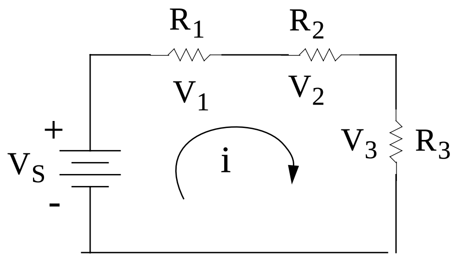
  - G (conductance) = 1 / R (resistance)
  - $G_{tot} = G_1 + G_2 + ... + G_n$
  - $= 1/R_1 * 1/R_2 + ... + 1/R_n$
  - So, $R_{tot} = 1 / G_{tot} = 1 / (1/R_1 + 1/R_2 + ... + 1/R_n)$
- ❑ Example, in case of 2 parallel resistors
  - $R_{tot} = (R_1 * R_2) / (R_1 + R_2)$

Series and Parallel DC Circuits

- ❑ Series connected:
  - All components see the same current
- ❑ Parallel connected:
  - All components see the same voltage drop
- ❑ Loop: A simple closed path in the circuit
- ❑ Brings us to Kirchhoff's Laws...

Kirchhoff's Voltage Law (KVL)

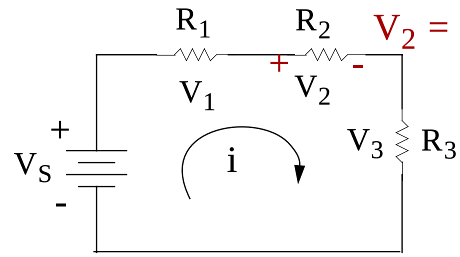
- Sum of voltages around a loop is 0



$$V_S = V_1 + V_2 + V_3 = I\,R_1 + I\,R_2 + I\,R_3 = I\,R_{tot}$$

Voltage Division

- Find V2, the voltage drop across R2



$V_2 = I\,R_2$

$$V_S = V_1 + V_2 + V_3 = I\,R_1 + I\,R_2 + I\,R_3 = I\,R_{tot}$$

$$I = V_S / (R_1 + R_2 + R_3)$$

$$\text{So } V_2 = \frac{R_2}{R_1 + R_2 + R_3} V_S$$

Voltage Division General Form

- Find voltage across any series-connected resistor

$$V_X = \frac{R_X}{R_{tot}} V_S$$

Resistance of resistor X

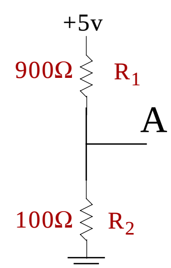
Voltage across resistor X

Total series resistance

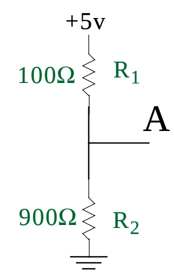
Total voltage

Example of Voltage Division

- Find voltage at point A with respect to GND



$$V_X = \frac{R_X}{R_1 + R_2} V$$



$V_1 = (900/1000)\ 5v = 4.5v$
$V_2 = (100/1000)\ 5v = 0.5v$

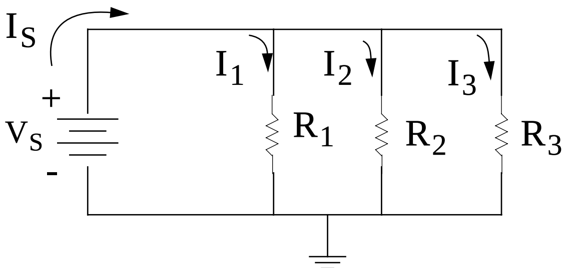
So, $V_{A\text{-}GND} = 0.5v$

$V_1 = (100/1000)\ 5v = 0.5v$
$V_2 = (900/1000)\ 5v = 4.5v$

So, $V_{A\text{-}GND} = 4.5v$

Kirchhoff's Current Law

- Sum of currents at any node in a circuit is 0



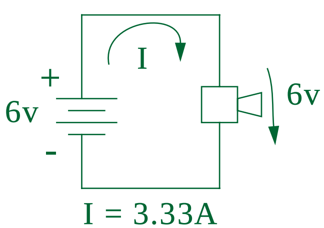
$$I_S = I_1 + I_2 + I_3$$

Example: Current limiting

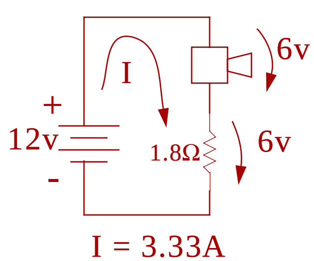
- Suppose you had a 20w horn from an old 6v car
- Want to put it in a new car with 12v system
- How do you make it work?
  - P = V I so if you increase the voltage without limiting current, the power goes up and the horn burns out
  - So, you need to limit the total current so that the horn sees the same current it was designed for
  - How? I = V / R, so if V goes up, R must also go up to keep current constant
  - So, what size resistor should you put in series with the horn to make this work?

Example: Current Limiting

- First compute how much current the horn would have seen in the 6v car
  - P = V I so I = P / V = 20w / 6v = 3.33A
- So, the series resistor should see the same current
  - R = 6v / 3.33A = 1.8Ω



Original System

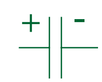


New System

Capacitors

Components that store electrical charge

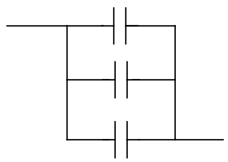
- Two conductors separated by an insulator
  - Accumulates charge on the plates



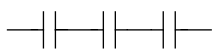
- SI unit is Farad
  - C (farads) = Q (Coulombs) / V (volts)
  - Capacitance of 1 farad means that putting +1 and -1 coulomb of charge on the plates results in a voltage difference of 1 volt
  - Or, a voltage of 1 volt forces 1 coulomb of charge on a capacitor
- Farad is *much* too large to be useful!
  - µF and pF are more common

Series and Parallel Capacitors

- Parallel connection: stores more charge
  - $C_{tot} = C_1 + C_2 + ... + C_n$

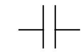


- Series connection: each plate steals charge from neighbor, so total capacitance is less
  - $C_{tot} = 1 / (1 / C_1 + 1 / C_2 + ... + 1/C_n)$



Charging a Capacitor

Each electron that comes in one lead "pushes" one electron from the other plate through the other lead

- Changing the voltage across a capacitor requires changing the charge stored on each plate, which requires current 
  - In a resistor, fixed current causes a fixed voltage drop: I = Q / t
  - In a capacitor, a fixed current causes a steadily increasing voltage drop as charge accumulates on the plates: i = dq / dt
  - We can't change voltage instantly across a capacitor because that would require infinite current!

q = cv

$$i = \frac{dq}{dt} = \frac{d}{dt}(cv)$$

But c is constant, so

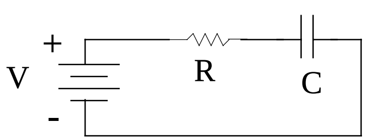
$$i = c\frac{dv}{dt}$$

$$\boxed{\frac{dv}{dt} = \frac{i}{c}}$$

Time to Charge a Capacitor

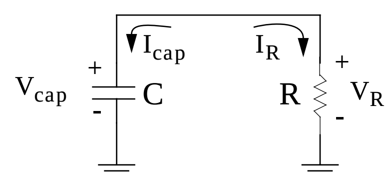
Initially very fast, then slows down exponentially

- Precise relationship depends on both R and C
- R (ohms) * C (farads) = what unit?
  - Answer: t (seconds)!
  - R = V / I
  - I = Q / t
  - C = Q / V
  - So, R C = (V) / (Q / t) * Q / V = (V)(t / Q)(Q / V) = t



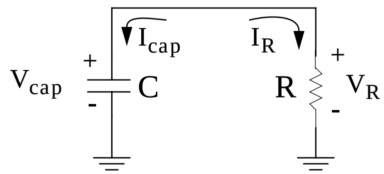
RC Time Constant

- Charging and discharging are exponential processes
  - Changing the voltage across a capacitor requires current
  - If the current flows through a resistor, it requires voltage across that resistor
  - If voltage decreases as the capacitor discharges, the current, and the rate of disharging decrease exponentially with time
  - Consider discharging a fully charged capacitor



Discharging a Capacitor

- According to Kirchhoff:
  - $V_R = V_{cap}$, and $I_R = -I_{cap}$
- Also:
  - $I_R = V_R / R$, and $dV_{cap} / dt = I_{cap} / C$
- Substituting, we get:
  - $dV_{cap} / dt = I_{cap} / C = -I_R / C = -V_{cap} / RC$
- Solving this differential equation:
  - $V_{cap}(t) = V_{cap}(0) * e^{-t/RC} = Vcc * e^{-t/RC}$



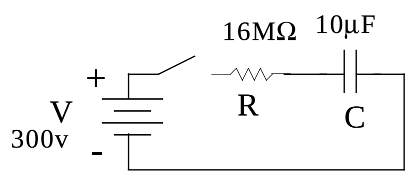
RC Time Constants

- General form:
  - $V_{(t)} = V_{(oo)} + [V_{(0)} - V_{(oo)}]\ e^{-t/RC}$
- Discharge from Vcc:
  - $V_{(t)} = Vcc\ e^{-t/RC}$
- Charge from GND:
  - $V_{(t)} = Vcc\ (1 - e^{-t/RC})$
- Short cut:
  - 99% of final charge or discharge in 5RC!

Example: RC Timer

Switch connects 300v, 16MΩ resistor, uncharged 10µF capacitor

- How long is switch closed if charge on capacitor is 10v?
  - Charging equation: $V_{(t)} = Vcc\ (1 - e^{-t/RC})$
  - $RC = 16{,}000{,}000\ \Omega * 10 \times 10^{-6}\ F = 160s$, $V_{(t)} = 10v$, $Vcc = 300v$
  - So, $10v = 300v\ (1 - e^{-t/160s})$
  - $300 - 10 = 300 * e^{-t/160}$
  - $290/300 = e^{-t/160}$
  - $\ln(290/300) = \ln(e^{-t/160})$
  - $\ln(290/300) = -t/160$
  - $t = -160 \ln(290/300)$
  - $t = 5.42s$



Energy Stored in a Capacitor

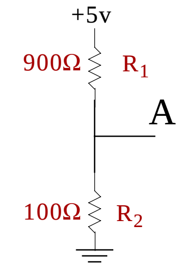
- Work must be done to separate charge
  - This energy is stored in the system and can be recovered by allowing the charge to come together again
  - I.e. a charged capacitor has potential energy equal to the work required to charge it
- Suppose at time t a charge of q(t) has been transferred from one plate to the other
  - The potential difference V(t) at this point is Q(t) / C
  - If an extra increment of charge dq is transferred, the extra work is dw = V dq = (q/c)dq
- So, the total work to move all the charge is $w = \int dw = \int_0^q (q/c)dq = 1/2\ q^2 / c$
- Since $q = cv$, $w = (1/2)\ cv^2$

Whew! Electronics Summary...

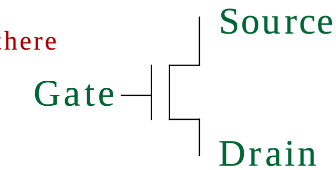
- Voltage is a measure of electrical potential energy
- Current is moving charge caused by voltage
- Resistance reduces current flow
  - Ohm's Law: V = I R
- Power is work over time
  - $P = V I = I^2 R$
- Capacitors store charge
  - It takes time to charge/discharge a capacitor
  - Time to charge/discharge is related exponentially to RC
  - It takes energy to charge a capacitor
  - Energy stored in a capacitor is $(1/2)\ C\ V^2$

How Does All This Relate To VLSI?

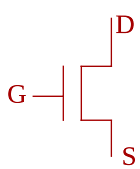
- Recall the voltage division example:
  - Consider what we could do if we had a device that we could switch from high resistance to low resistance
  - We could use it to force A high or low depending on the relative resistance of the elements



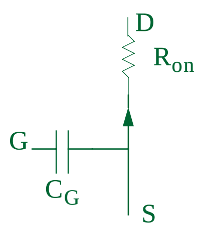
- This is a transistor
  - Specifically a CMOS FET
  - Complementary Metal-Oxide Semiconductor Field Effect Transistor
  - If voltage on Gate is high, then there is a low-resistance between Source and Drain, otherwise it's a very high-resistance



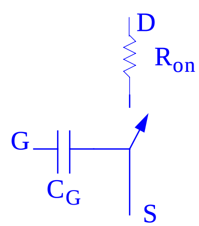
Electrical Model of a CMOS Transistor



Switch Level Model



Switch is closed if Gate voltage is high

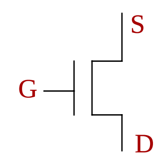


Switch is open if Gate voltage is low

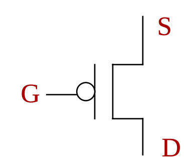
R_{on} = Some resistance in FET itself
C_G = Capacitance of the gate

Two Types of CMOS Transistors

- N-type transistor
  - High voltage on Gate connects Source to Drain
  - Passes 0 well, passes 1 poorly

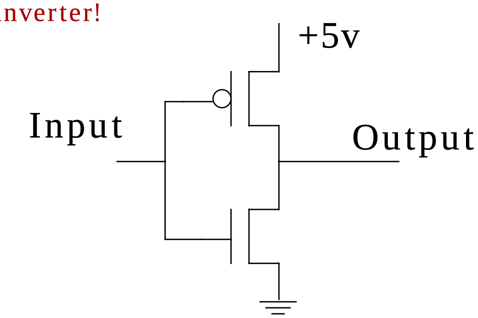


- P-type transistor
  - Low voltage on Gate connects Source to Drain
  - Passes 1 well, passes 0 poorly



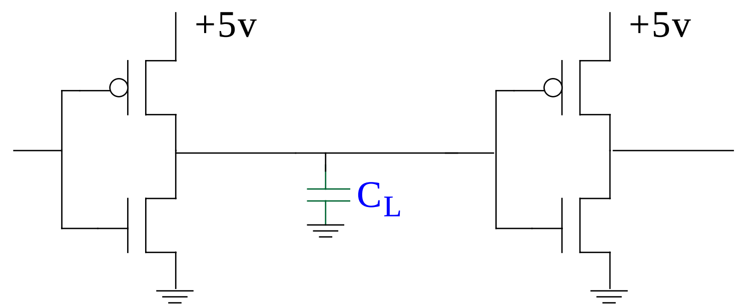
CMOS Inverter

- Consider this connection of transistors
  - If input is at a high voltage, output is low
  - If input is at a low voltage, output is high
- By changing the resistances, it becomes one of two different voltage dividers
  - It's a voltage inverter!

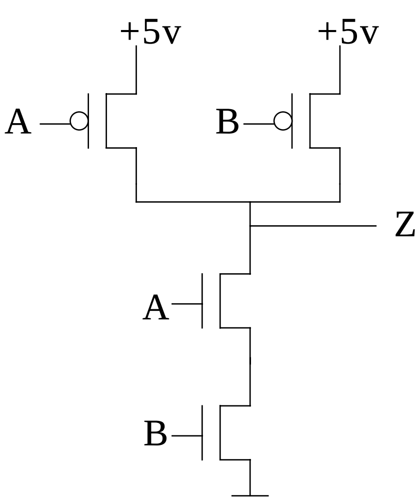


Timing Issues in CMOS

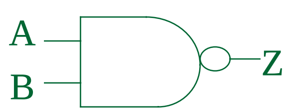
- Recall that it takes time to charge capacitors
- Recall that the gate of a transistor looks like a capacitor
- Wires have resistance and capacitance also!



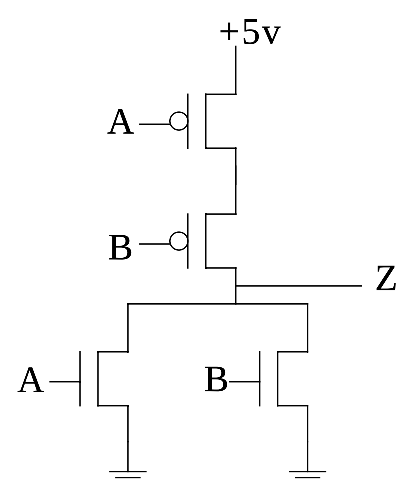
CMOS NAND Gate



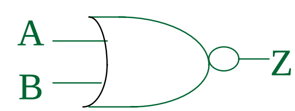
A B	Z
00	1
01	1
10	1
11	0



CMOS NOR Gate



A B	Z
00	1
01	0
10	0
11	0



CMOS Power Consumption

- Power is consumed in CMOS by charging and discharging capacitors
  - Note that there no static power dissipation in CMOS
  - There's never a DC path to ground
- Good news:
  - You're not consuming power unless you're switching
- Bad news:
  - Switching activity is caused by clock, which is going faster and faster
- If the first-order power effect is capacitor charging/discharging, and the clock causes this:

$$P = (1/2)\ C\ V^2\ f$$

Is That All There is to VLSI?

- We've got NAND, NOR, and INV gates
  - With those we should be able to build anything
- We've also got some idea of why things can't go infinitely fast
  - We've got to keep charging and discharging those darn capacitors!
- We've got some idea of where and why power is consumed
  - We've got to keep charging and discharging those darn capacitors!
- And a hint why power supply voltages are getting lower
  - $P = (1/2)CV^2f$, Which one would you optimize first?

Conclusions

- That's about all I have the stamina for
  - I'll be a little surprised if we even make it through all the slides to the end!
- A little knowledge of basic electronics can explain a lot about computer hardware
- A little more knowledge about VLSI could explain even more!
  - But that's a subject for another lecture!