Electronics for Computer Scientists

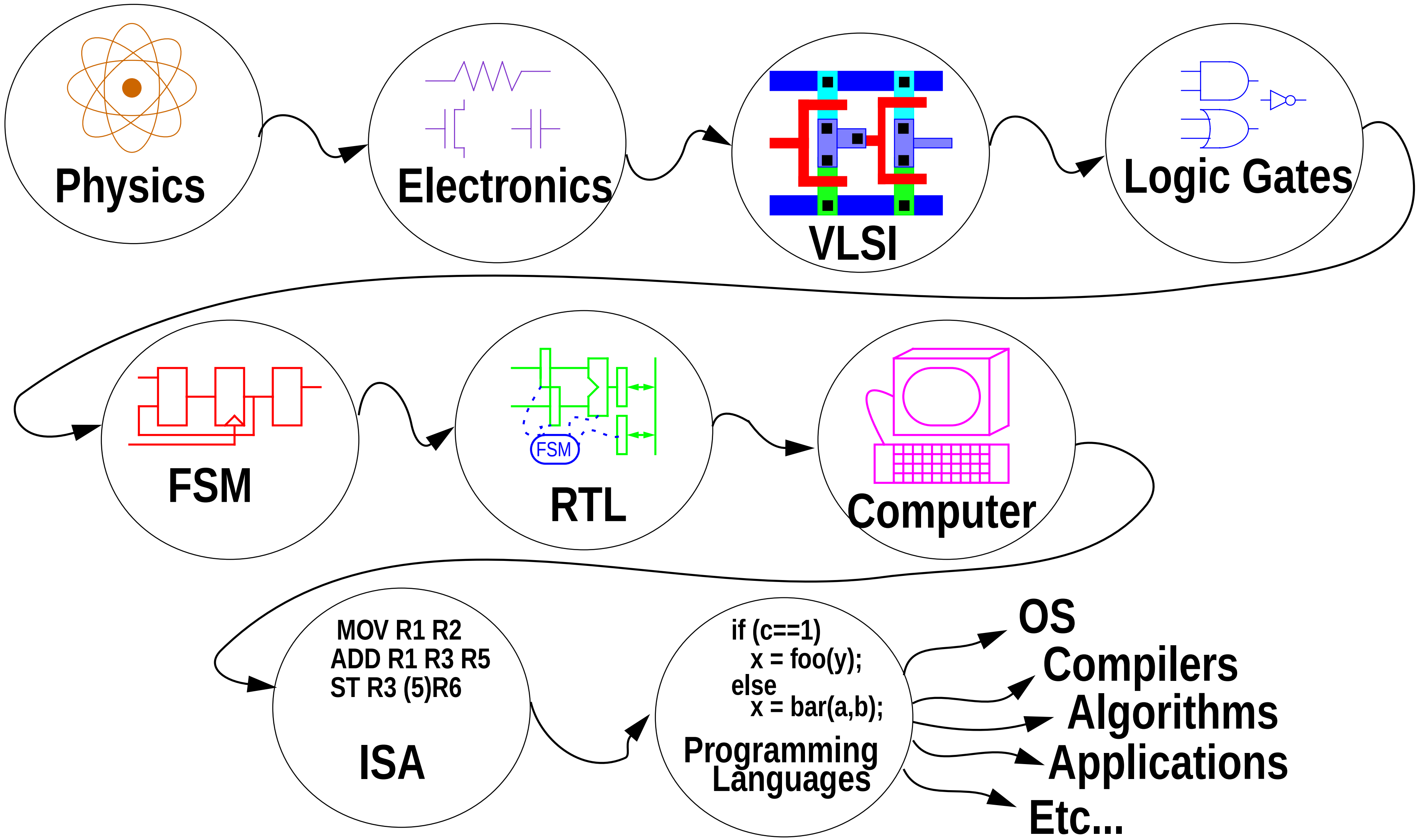
Ohm's Law to VLSI

Erik Brunvand

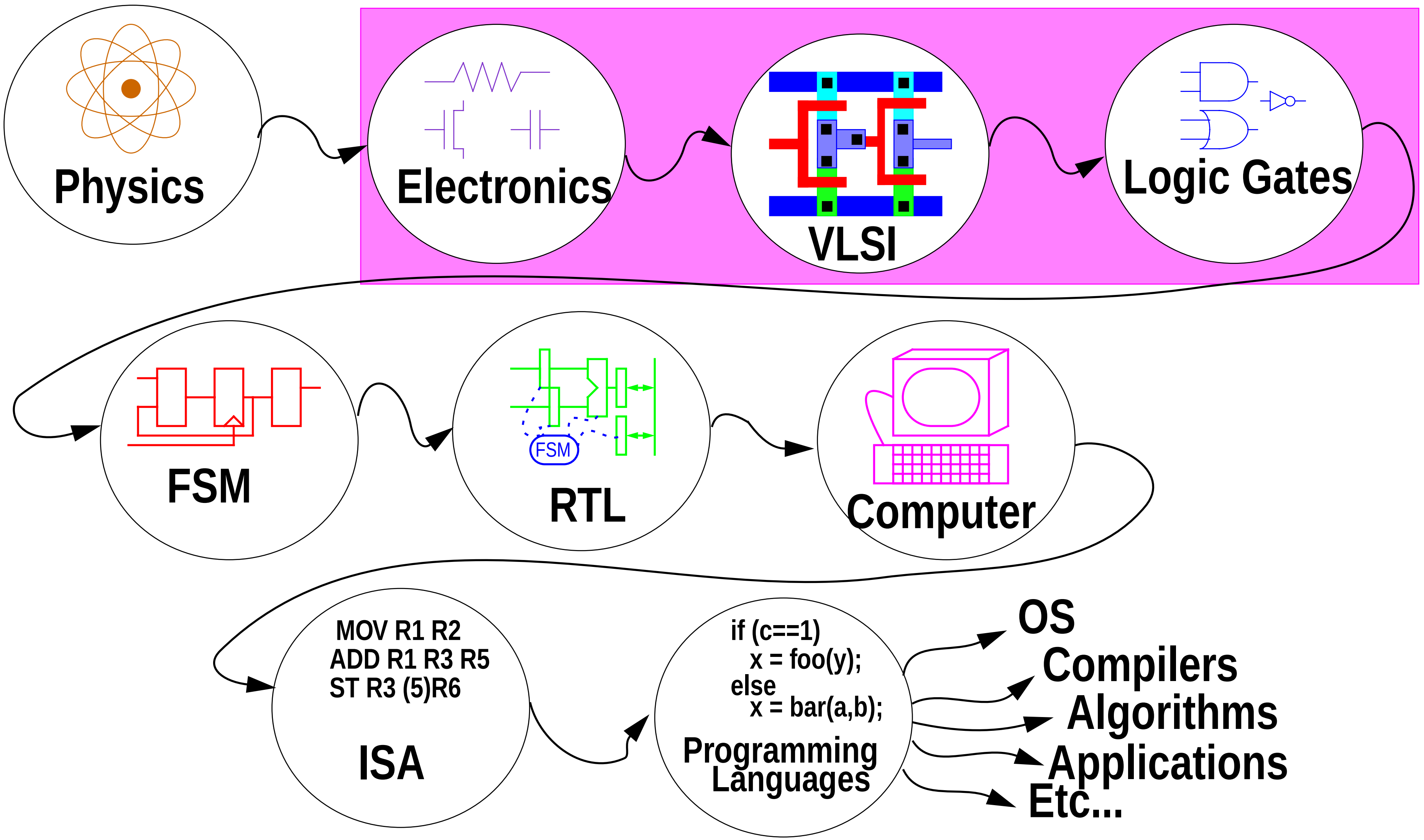
Why do we want to know?

- It's important to know something about computing hardware
- If only to not sound like a dummy…
  - How much power does your PC draw?
  - Why does your laptop only last a hour on a battery, but your watch lasts 2 years?
  - Why does a faster processor burn more power?
  - 700MHz is pretty fast. What are the issues in making things go faster?
  - How are logic gates built? How do they work?
  - How are logic gates used to build computing systems?
- It also lets you understand and appreciate limitations and advances in hardware

The Big Picture



This Talk


Physics
Electronics
VLSI
Logic Gates
FSM
RTL
FSM
Computer
MOV R1 R2
ADD R1 R3 R5
ST R3 (5)R6
ISA
if (c==1)
x = foo(y);
else
x = bar(a,b);
Programming Languages
OS
Compilers
Algorithms
Applications
Etc...

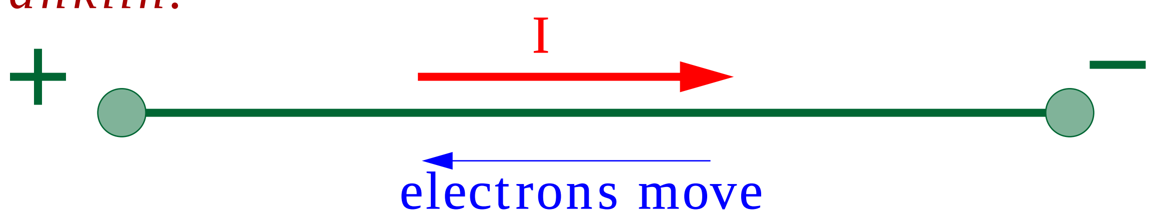
Electric Charge

- Atomic-level property
  - Positive charge = Proton
  - Negative charge = Electron
- Charges produce force against each other
  - Like charges repel
  - Different charges attract
- SI unit of charge is Coulomb (Q, q are quantity symbols)
  - Charge on electron is -1.602×10^{-19} Coulombs
  - 6.241×10^{18} electrons = 1 Coulomb

Electric Current

Results from charge moving in a conductor

- SI unit of current is Ampere, Amp, A (I, i are quantity symbols)
  - 1 Amp is 1 Coulomb of charge passing a point in 1 second
  - I (Amperes) = Q (Coulombs) / t (seconds)
- Current has a direction: it flows from positive to negative points (positive current)
  - But, electrons are really the things that move in the conductor
  - And, they move from negative to positive
  - So, the electrons move in the opposite direction as current flow
  - *Blame Ben Franklin!*



Voltage

Difference in electrical potential at two points in a circuit

- A measure of how much work is involved in moving charge between those points
  - W (joules) = F (newtons) * s (meters)
- Energy is the capacity to do work.
  - Potential energy is energy something has because of position
  - Voltage difference is a potential difference
- Voltage is the energy that causes current to flow
  - Current flows from higher potential to lower potential

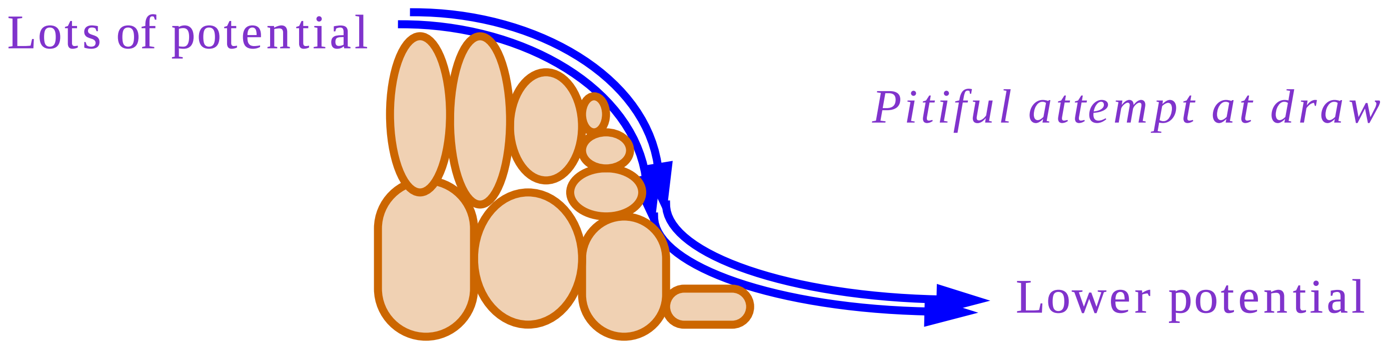
Voltage is Relative

Measured relative to two points in a system

- 1 Volt is the work required to move 1 Coulomb of charge from one point to another
  - V_{a-b} (volts) = W (joules) / Q (Coulombs)
- Raising the voltage of one Coulomb of charge by 1 volt takes 1 joule of energy...
- One point is arbitrarily called 0v or Ground (GND)
  - Which means that voltage can easily be negative with respect to that arbitrary point

Water Analogy

- Current flow = water flow
- Amount of current = how much water
- Voltage = potential energy of the water
  - 0v = stagnant pool of water, no flow
  - Small voltage = tiny waterfall, not much energy
  - Large voltage = large waterfall, lots of energy
  - Negative voltage = dig a hole under the pond
- More water analogy later....



Pitiful attempt at drawing a waterfall...

Power

The rate at which something produces or consumes energy

- P (watts) = W (joules) / t (seconds)

$$P\ (\text{watts}) = \frac{W\ (\text{joules})}{Q\ (\text{coulombs})} * \frac{Q\ (\text{coulombs})}{t\ (\text{seconds})}$$

$$P\ (\text{watts}) = V\ (\text{volts}) * I\ (\text{Amperes})$$

Example

- How much current flows in a light bulb from a steady movement of 10^{22} electrons in 1 hour?

$$\frac{10^{22}\ \text{electrons}}{1\text{h}} * \frac{1\text{h}}{3600\text{s}} * \frac{-1.602\text{x}10^{-19}\ \text{C}}{1\ \text{electron}} = -0.445\text{C/s}$$

$$= -0.445\text{A}$$

Example

- How much current does a 1200w toaster draw from a 120v power connection?

$$P = V I$$

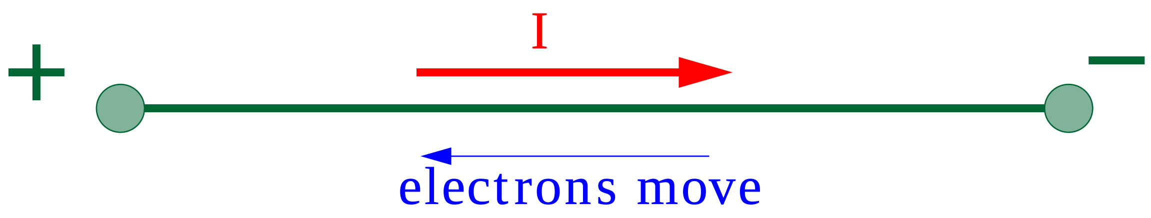
$$I = P/V = 1200w/120v = 10A$$

How fast do electrons move?

What is the "drift velocity" of an electron?

- Example: 14 gauge copper wire, 10A current
  - Copper wire has 1.38×10^{24} free electrons/in^3
  - 14 gauge cross section is $3.23/10^{-3}$ in^2
  - Electron velocity is (current)/(area * electron density)

Electrical impulse moves at 2.998×10^8 m/s
(i.e. close to speed of light)



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  - 14 gauge cross section is $3.23/10^{-3}\ in^2$
  - Electron velocity is (current)/(area * electron density)

$$\text{velocity} = \frac{10C}{1s} * \frac{1}{3.23 \times 10^{-3} in^2} * \frac{1 in^3}{1.38 \times 10^{24}\ \text{electrons}}$$

$$= \frac{10C}{1s} * \frac{1}{3.23 \times 10^{-3} in^2} * \frac{1 in^3}{1.38 \times 10^{24}\ \text{electrons}} * \frac{0.0254m}{1in} * \frac{1\ \text{electron}}{-1.602 \times 10^{-19} C}$$

= -3.56x10-4 m/s * 3600s/h = -1.28m/h (Very slow!!!)

Resistance

The property that opposes or resists current flow

- Water analogy:
  - friction of water in a small pipe
- Electronics:
  - Electrons collide with conductor atoms and lose energy in the form of heat
- Current is proportional to applied voltage
  - Unit is the Ohm, symbol is Ω
  - Ohm's Law: I (amps) = V (volts) / R (Ohms)
  - I = V/R or V = I R

Resistance of Materials

Proportional to length
inversely proportional to cross-section area

- Big Pipe = less force (voltage) required to push water (current) through
- Little Pipe = more force (voltage) required to force the same amount of current through
  - Resistance = ρ (L / A) where ρ is "resistivity"in Ωm

Material	Resistivity	Material	Resistivity
Silver	1.64×10^{-8}	Nichrome	100×10^{-8}
Copper	1.72×10^{-8}	Silicon	2500
Aluminum	2.83×10^{-8}	Quartz	10^{17}

(note, this property is measurable over 25 orders of magnitude!)

Example

- Given a 240v heating element in a stove that has 24 Ω resistance, what fuse to use?
  - Fuse must be able to carry the current of the heating element
  - $I = V / R = 240v / 24\Omega = 10A$
- How much power does this heating element dissipate?
  - Recall $P = V I$, and $V = I R$, so $P = I^2 R$
  - So $P = 10^2 * 24W = 2400 W$

Example

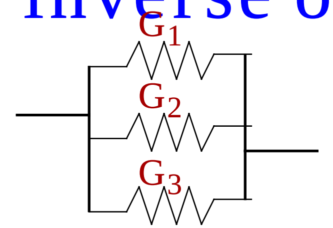
- What is the resistance of an Al wire 1000m long with diameter 1.626mm?
  - Cross sectional area = Πr^2, r=d/2 = 0.813×10^{-3}m
  - R (ohms) = ρ (L / A)

$$= \frac{(2.83 \times 10^{-8} \Omega m)(1000 m)}{\Pi (0.813 \times 10^{-3} m)^2} = 13.6\Omega$$

Series and Parallel Connections of Resistors

- Resistors in series = more total resistance
  - $R_{tot} = R_1 + R_2 + \dots + R_n$

- Resistors in parallel = less total resistance
- Think about conductance as the inverse of resistance



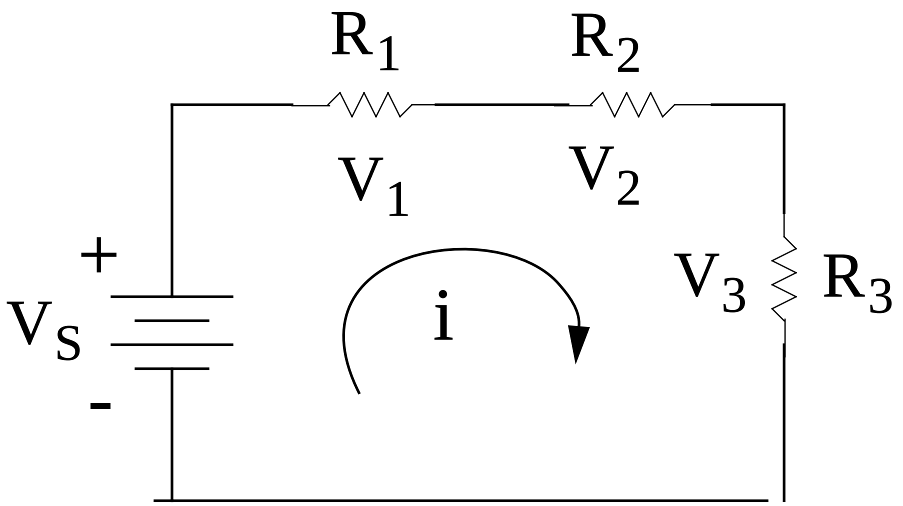
  - G (conductance) = 1 / R (resistance)
  - $G_{tot} = G_1 + G_2 + \dots + G_n$
  - $= 1/R_1 * 1/R_2 + \dots + 1/R_n$
  - So, $R_{tot} = 1 / G_{tot} = 1 / (1/R_1 + 1/R_2 + \dots + 1/R_n)$
- Example, in case of 2 parallel resistors
  - $R_{tot} = (R_1 * R_2) / (R_1 + R_2)$

Series and Parallel DC Circuits

- Series connected:
  - All components see the same current
- Parallel connected:
  - All components see the same voltage drop
- Loop: A simple closed path in the circuit
- Brings us to Kirchhoff's Laws…

Kirchhoff's Voltage Law (KVL)

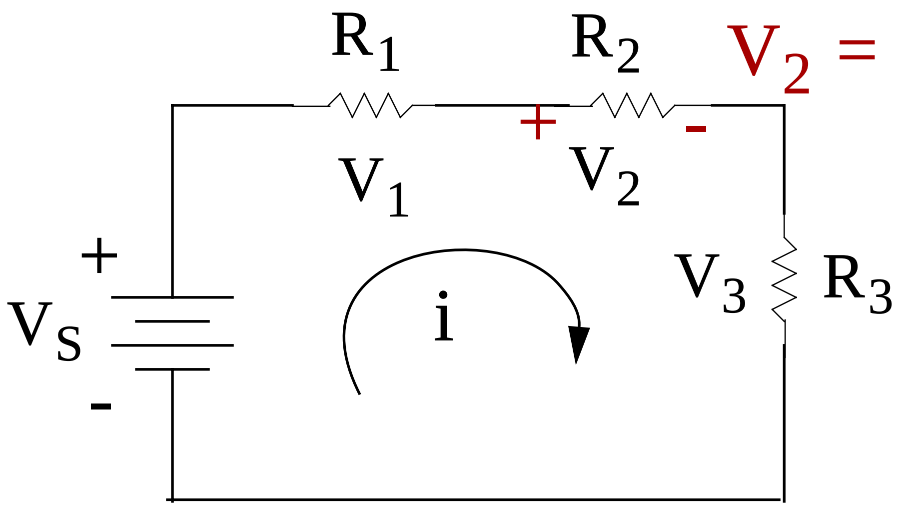
- Sum of voltages around a loop is 0



$$V_S = V_1 + V_2 + V_3 = I\ R_1 + I\ R_2 + I\ R_3 = I\ R_{tot}$$

Voltage Division

- Find V2, the voltage drop across R2



$$V_2 = I\,R_2$$

$$V_S = V_1 + V_2 + V_3 = I\,R_1 + I\,R_2 + I\,R_3 = I\,R_{tot}$$

$$I = V_S / (R_1 + R_2 + R_3)$$

$$\text{So } V_2 = \frac{R_2}{R_1 + R_2 + R_3} V_S$$

Voltage Division General Form

- Find voltage across any series-connected resistor

Resistance of resistor X

$$V_X = \frac{R_X}{R_{tot}} V_S$$

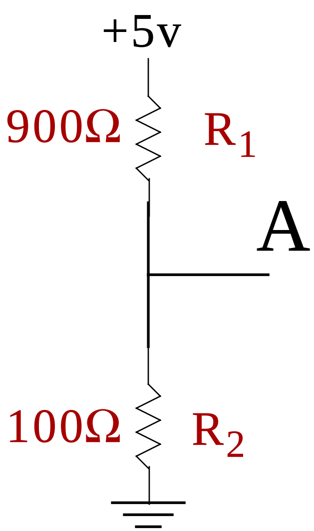
Voltage across resistor X

Total series resistance

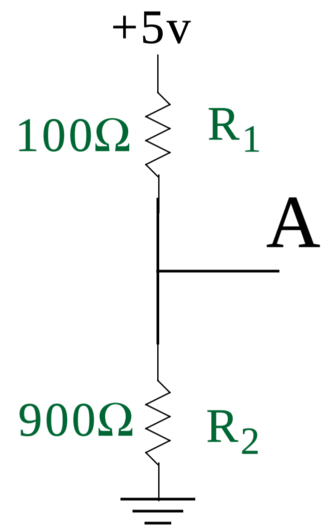
Total voltage

Example of Voltage Division

- Find voltage at point A with respect to GND



$$V_X = \frac{R_X}{R_1 + R_2} V$$



V_1 = (900/1000) 5v = 4.5v
V_2 = (100/1000) 5v = 0.5v

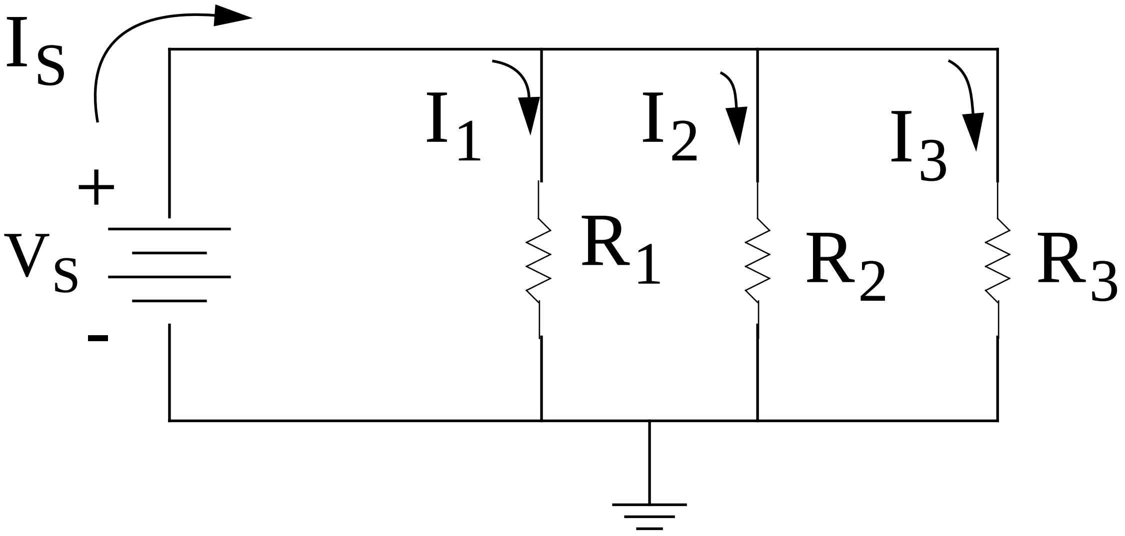
So, $V_{A\text{-}GND}$ = 0.5v

V_1 = (100/1000) 5v = 0.5v
V_2 = (900/1000) 5v = 4.5v

So, $V_{A\text{-}GND}$ = 4.5v

Kirchhoff's Current Law

- Sum of currents at any node in a circuit is 0



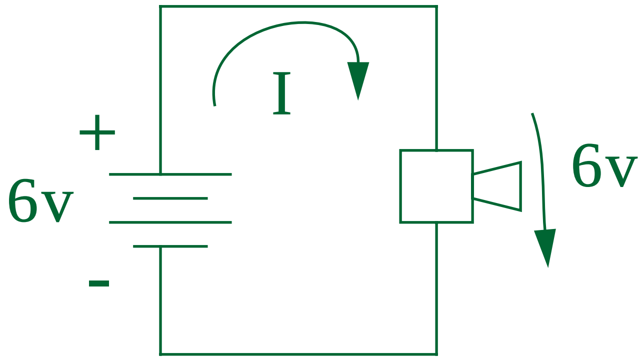
$$I_S = I_1 + I_2 + I_3$$

Example: Current limiting

- Suppose you had a 20w horn from an old 6v car
- Want to put it in a new car with 12v system
- How do you make it work?
  - P = V I so if you increase the voltage without limiting current, the power goes up and the horn burns out
  - So, you need to limit the total current so that the horn sees the same current it was designed for
  - How? I = V / R, so if V goes up, R must also go up to keep current constant
  - So, what size resistor should you put in series with the horn to make this work?

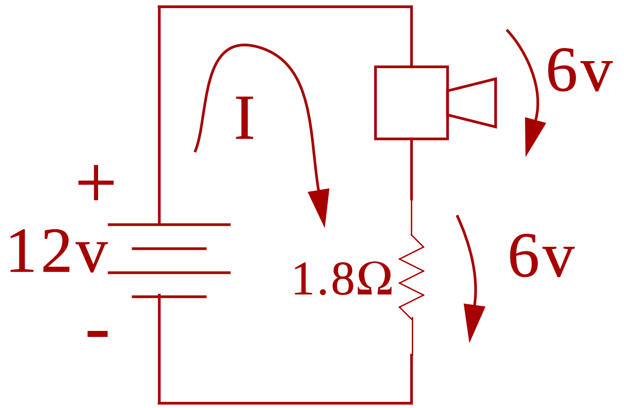
Example: Current Limiting

- First compute how much current the horn would have seen in the 6v car
  - P = V I so I = P / V = 20w / 6v = 3.33A
- So, the series resistor should see the same current
  - R = 6v / 3.33A = 1.8Ω



I = 3.33A

Original System



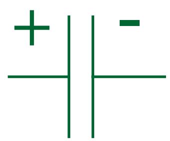
I = 3.33A

New System

Capacitors

Components that store electrical charge

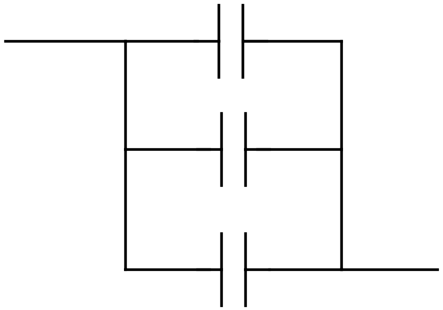
- Two conductors separated by an insulator
  - Accumulates charge on the plates



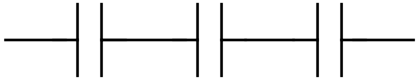
- SI unit is Farad
  - C (farads) = Q (Coulombs) / V (volts)
  - Capacitance of 1 farad means that putting +1 and -1 coulomb of charge on the plates results in a voltage difference of 1 volt
  - Or, a voltage of 1 volt forces 1 coulomb of charge on a capacitor
- Farad is *much* too large to be useful!
  - μF and pF are more common

Series and Parallel Capacitors

- Parallel connection: stores more charge
  - $C_{tot} = C_1 + C_2 + \ldots + C_n$

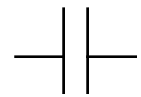


- Series connection: each plate steals charge from neighbor, so total capacitance is less
  - $C_{tot} = 1 / (1 / C_1 + 1 / C_2 + \ldots + 1/C_n)$



Charging a Capacitor

Each electron that comes in one lead "pushes" one electron from the other plate through the other lead

- Changing the voltage across a capacitor requires changing the charge stored on each plate, which requires current 
  - In a resistor, fixed current causes a fixed voltage drop: I = Q / t
  - In a capacitor, a fixed current causes a steadily increasing voltage drop as charge accumulates on the plates: i = dq / dt
  - We can't change voltage instantly across a capacitor because that would require infinite current!

$q = cv$

$$i = \frac{dq}{dt} = \frac{d}{dt}(cv)$$

But c is constant, so

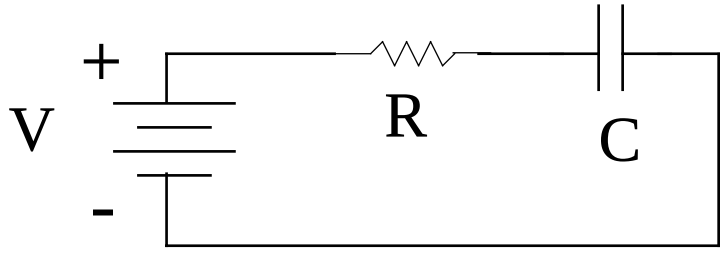
$$i = c\frac{dv}{dt}$$

$$\boxed{\frac{dv}{dt} = \frac{i}{c}}$$

Time to Charge a Capacitor

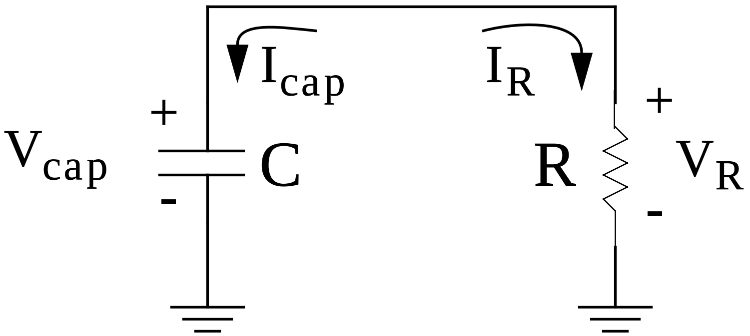
Initially very fast, then slows down exponentially

- Precise relationship depends on both R and C
- R (ohms) * C (farads) = what unit?
  - Answer: t (seconds)!
  - R = V / I
  - I = Q / t
  - C = Q / V
  - So, R C = (V) / (Q / t) * Q / V = (V)(t / Q)(Q / V) = t



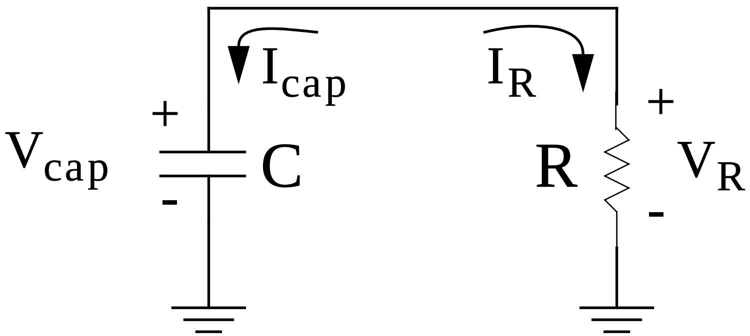
RC Time Constant

- Charging and discharging are exponential processes
  - Changing the voltage across a capacitor requires current
  - If the current flows through a resistor, it requires voltage across that resistor
  - If voltage decreases as the capacitor discharges, the current, and the rate of disharging decrease exponentially with time
  - Consider discharging a fully charged capacitor



Discharging a Capacitor

- According to Kirchhoff:
  - $V_R = V_{cap}$, and $I_R = -I_{cap}$
- Also:
  - $I_R = V_R / R$, and $dV_{cap} / dt = I_{cap} / C$
- Substituting, we get:
  - $dV_{cap} / dt = I_{cap} / C = -I_R / C = -V_{cap} / RC$
- Solving this differential equation:
  - $V_{cap}(t) = V_{cap}(0) * e^{-t/RC} = Vcc * e^{-t/RC}$



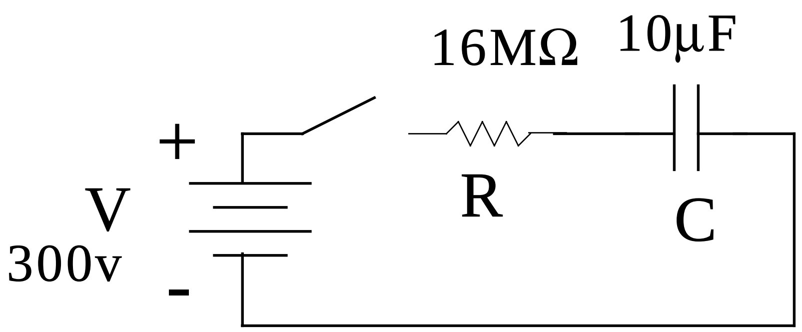
RC Time Constants

- General form:
  - $V_{(t)} = V_{(oo)} + [V_{(0)} - V_{(oo)}]\, e^{-t/RC}$
- Discharge from Vcc:
  - $V_{(t)} = Vcc\, e^{-t/RC}$
- Charge from GND:
  - $V_{(t)} = Vcc\,(1 - e^{-t/RC})$
- Short cut:
  - 99% of final charge or discharge in 5RC!

Example: RC Timer

Switch connects 300v, 16MΩ resistor, uncharged 10μF capacitor

- How long is switch closed if charge on capacitor is 10v?
  - Charging equation: $V_{(t)} = Vcc\,(1 - e^{-t/RC})$
  - $RC = 16{,}000{,}000\ \Omega * 10 \times 10^{-6}\ F = 160s$, $V_{(t)} = 10v$, $Vcc = 300v$
  - So, $10v = 300v\,(1 - e^{-t/160s})$
  - $300 - 10 = 300 * e^{-t/160}$
  - $290/300 = e^{-t/160}$
  - $\ln(290/300) = \ln(e^{-t/160})$
  - $\ln(290/300) = -t/160$
  - $t = -160 \ln(290/300)$
  - $t = 5.42s$



Energy Stored in a Capacitor

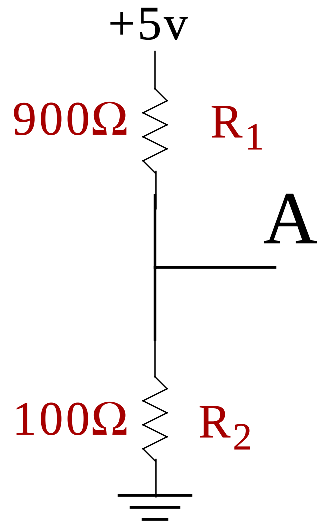
- Work must be done to separate charge
  - This energy is stored in the system and can be recovered by allowing the charge to come together again
  - I.e. a charged capacitor has potential energy equal to the work required to charge it
- Suppose at time t a charge of q(t) has been transferred from one plate to the other
  - The potential difference V(t) at this point is Q(t) / C
  - If an extra increment of charge dq is transferred, the extra work is $dw = V\,dq = (q/c)dq$
- So, the total work to move all the charge is $w = \int dw = \int_0^q (q/c)dq = 1/2\ q^2 / c$
- Since $q = cv$, $w = (1/2)\ cv^2$

Whew! Electronics Summary...

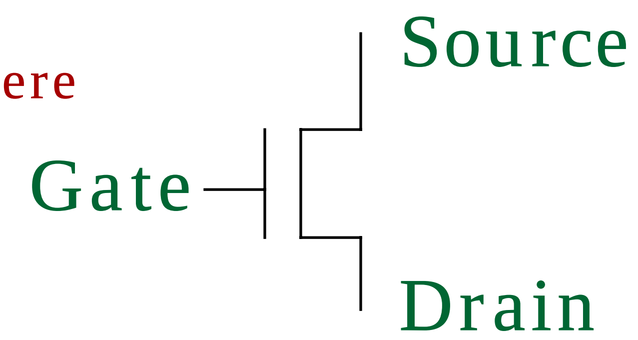
- Voltage is a measure of electrical potential energy
- Current is moving charge caused by voltage
- Resistance reduces current flow
  - Ohm's Law: V = I R
- Power is work over time
  - $P = V I = I^2 R$
- Capacitors store charge
  - It takes time to charge/discharge a capacitor
  - Time to charge/discharge is related exponentially to RC
  - It takes energy to charge a capacitor
  - Energy stored in a capacitor is $(1/2)\, C V^2$

How Does All This Relate To VLSI?

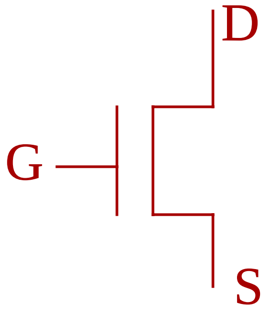
- Recall the voltage division example:
    - Consider what we could do if we had a device that we could switch from high resistance to low resistance
    - We could use it to force A high or low depending on the relative resistance of the elements



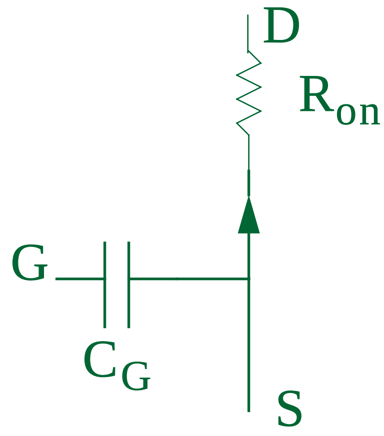
- This is a transistor
    - Specifically a CMOS FET
    - Complementary Metal-Oxide Semiconductor Field Effect Transistor
    - If voltage on Gate is high, then there is a low-resistance between Source and Drain, otherwise it's a very high-resistance



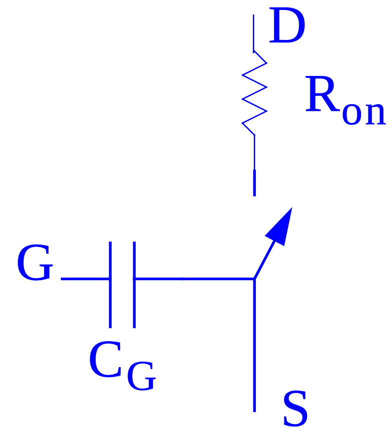
Electrical Model of a CMOS Transistor



Switch Level Model



Switch is closed if Gate voltage is high



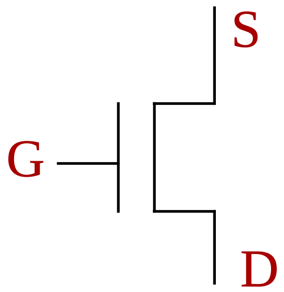
Switch is open if Gate voltage is low

R_{on} = Some resistance in FET itself

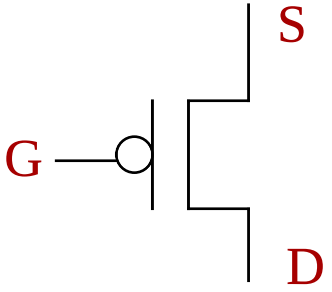
C_G = Capacitance of the gate

Two Types of CMOS Transistors

- N-type transistor
  - High voltage on Gate connects Source to Drain
  - Passes 0 well, passes 1 poorly

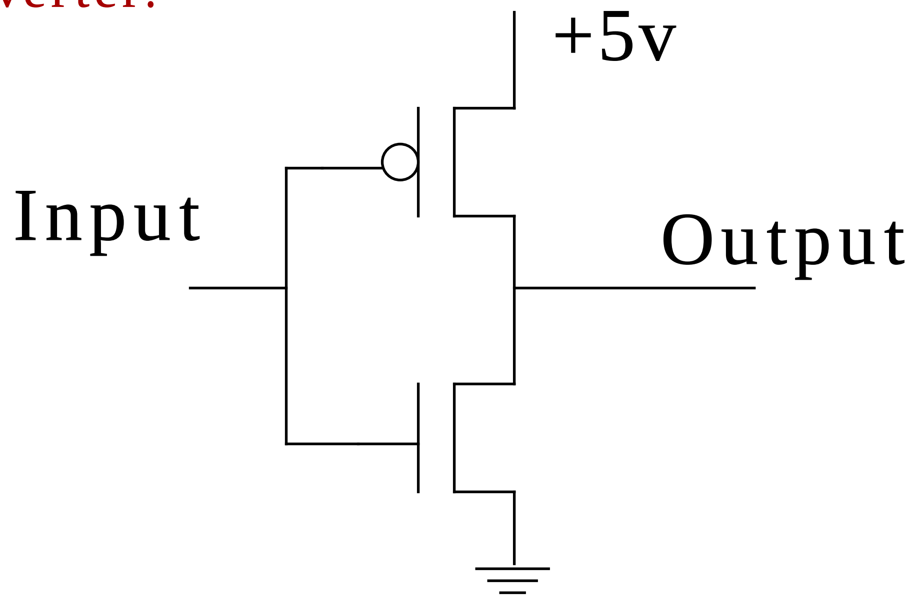


- P-type transistor
  - Low voltage on Gate connects Source to Drain
  - Passes 1 well, passes 0 poorly



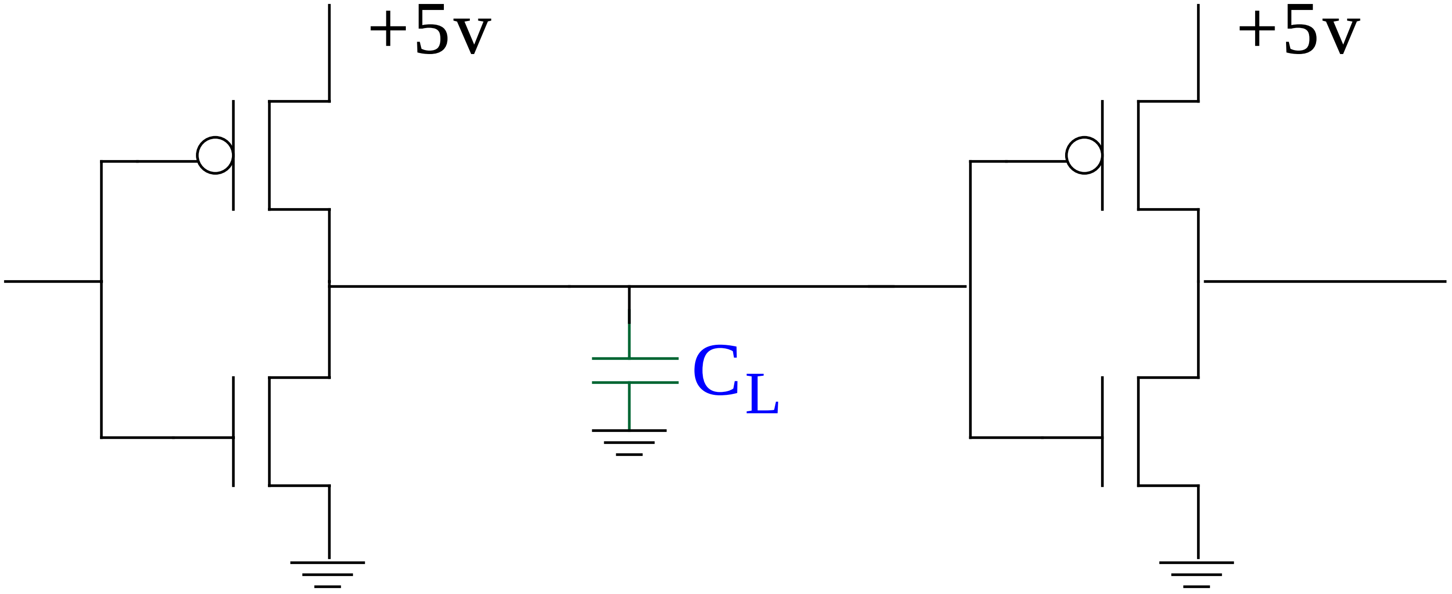
CMOS Inverter

- Consider this connection of transistors
  - If input is at a high voltage, output is low
  - If input is at a low voltage, output is high
- By changing the resistances, it becomes one of two different voltage dividers
  - It's a voltage inverter!

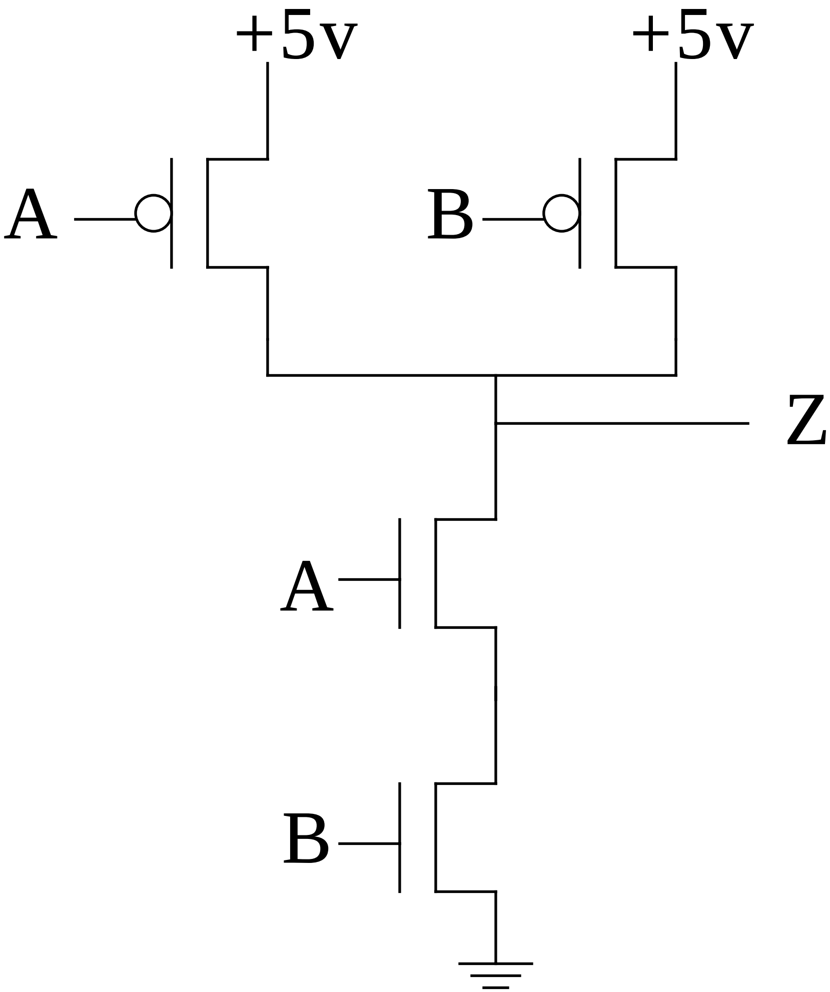


Timing Issues in CMOS

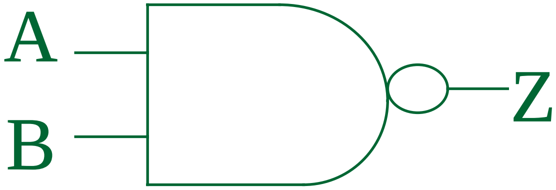
- Recall that it takes time to charge capacitors
- Recall that the gate of a transistor looks like a capacitor
- Wires have resistance and capacitance also!



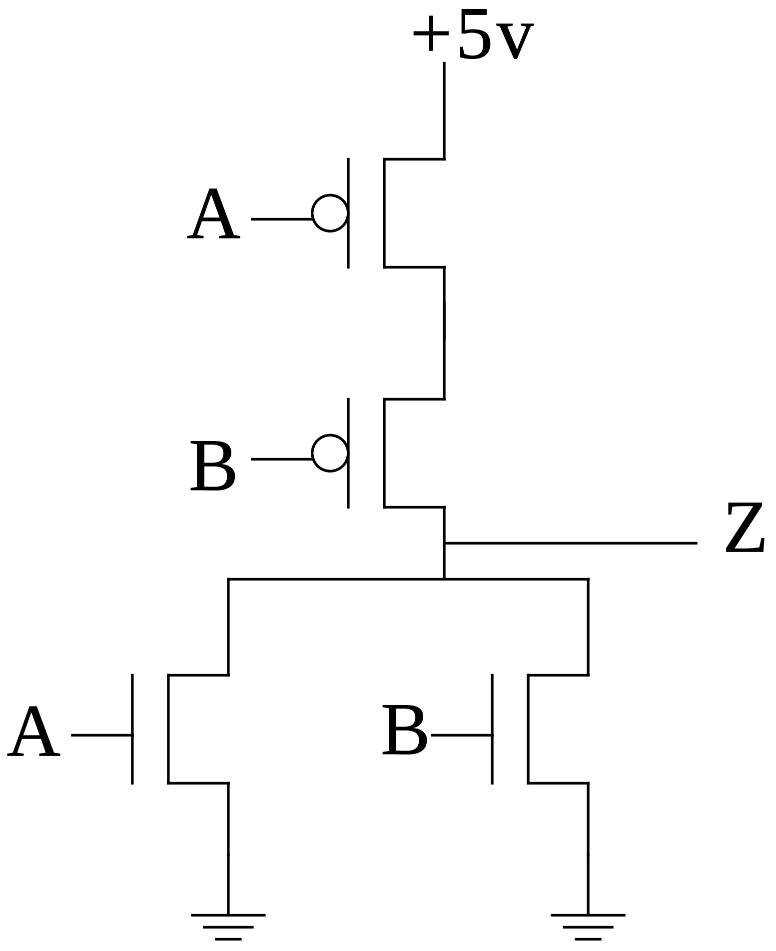
CMOS NAND Gate



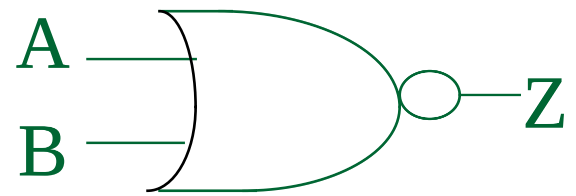
A B	Z
00	1
01	1
10	1
11	0



CMOS NOR Gate



A B	Z
00	1
01	0
10	0
11	0



CMOS Power Consumption

- Power is consumed in CMOS by charging and discharging capacitors
  - Note that there no static power dissipation in CMOS
  - There's never a DC path to ground
- Good news:
  - You're not consuming power unless you're switching
- Bad news:
  - Switching activity is caused by clock, which is going faster and faster
- If the first-order power effect is capacitor charging/discharging, and the clock causes this:

$$P = (1/2)\, C\, V^2\, f$$

Is That All There is to VLSI?

- We've got NAND, NOR, and INV gates
  - With those we should be able to build anything
- We've also got some idea of why things can't go infinitely fast
  - We've got to keep charging and discharging those darn capacitors!
- We've got some idea of where and why power is consumed
  - We've got to keep charging and discharging those darn capacitors!
- And a hint why power supply voltages are getting lower
  - $P = (1/2)CV^2f$, Which one would you optimize first?

Conclusions

- ❑ That's about all I have the stamina for
  - I'll be a little surprised if we even make it through all the slides to the end!
- ❑ A little knowledge of basic electronics can explain a lot about computer hardware
- ❑ A little more knowledge about VLSI could explain even more!
  - But that's a subject for another lecture!