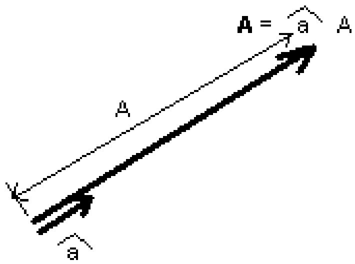
<u>ECE 3300 : Vector Analysis and Coordinate Systems</u>

Why learn vectors? Because most of the physical world (including waves!) MOVES in a 3D space. This can best be described by vectors.

Also, fields and waves are described as vectors. Knowing that the E field is __ amount means very little. We also want to know what direction the field is in. (ie. how the charges are separated.) So far we have studied voltage and current “waves” in a TEM transmission line. The only reason we did not have to account for the vector nature of these fields is that we knew exactly what direction they were moving, and that they were perpendicular to the direction of propagation. Now we are going to expand our understanding to general fields, which must be described by vectors.



Magnitude: specifies length of vector
$A = |\mathbf{A}|$

Unit Vector: specifies direction
$\mathbf{a}$ = vector in direction of $\mathbf{A}$ with length = 1 (UNIT length)
$\mathbf{a} = \mathbf{A} / |\mathbf{A}| = \mathbf{A} / A$

What is still missing? ORIGIN of vector

Base Vectors:
set of perpendicular (orthogonal) UNIT vectors which can be used to define any other vector.
ie. **x,y,z**

$\mathbf{A} = \mathbf{x} A_x + A_y \mathbf{y} + A_z \mathbf{z} = (A_x , A_y , A_z)$ shorthand notation

$A = |\mathbf{A}| = (A_x^2 + A_y^2 + A_z^2)^{1/2}$

$\mathbf{a} = \mathbf{A} / |\mathbf{A}| = \mathbf{A} / A = (\mathbf{x} A_x + A_y \mathbf{y} + A_z \mathbf{z}) / (A_x^2 + A_y^2 + A_z^2)^{1/2}$

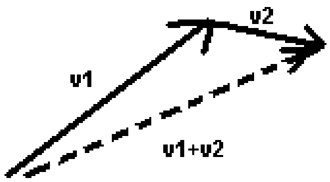
Vector Equality: (NOT identicalness)
$\mathbf{A} = \mathbf{B}$ iff $A_x = B_x$; $A_y = A_y$; $A_z = B_z$
Magnitudes and directions are the same.
But origins may not be.
Two parallel vectors (rectangular coordinates) are equal.

Vector Addition/Subtraction

A = $\mathbf{x} A_x + A_y \mathbf{y} + A_z \mathbf{z}$

B = $\mathbf{x} B_x + B_y \mathbf{y} + B_z \mathbf{z}$

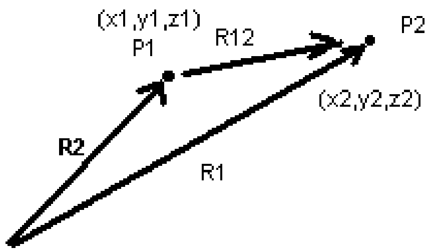
A+B = B+A = $(A_x + B_x)\,\mathbf{x} + (A_y + B_y)\,\mathbf{y} + (A_z + B_z)\mathbf{z}$



Distance Vector:

Describes VECTOR distance between two points

(magnitude and direction to travel from one point P1 to another P2)



R1 = $x1\,\mathbf{x} + y1\,\mathbf{y} + z1\,\mathbf{z}$

R2 = $x2\,\mathbf{x} + y2\,\mathbf{y} + z2\,\mathbf{z}$

R12 = R2 – R1 = $(x2 - x1)\,\mathbf{x} + (y2 - y1)\,\mathbf{y} + (z2 - z1)\,\mathbf{z}$

Distance between P1 and P2 = |**R12**|

Vector Multiplication:

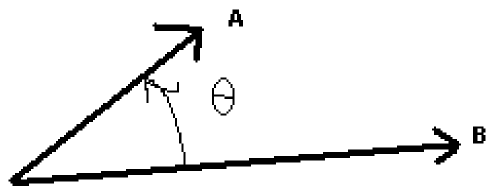
(a) **Multiplying by a scalar**

(making the vector magnitude larger)

$\mathbf{B} = k\mathbf{A} = kA_x\,\mathbf{x} + kA_y\,\mathbf{y} + kA_z\,\mathbf{z}$

(b) Scalar or Dot Product

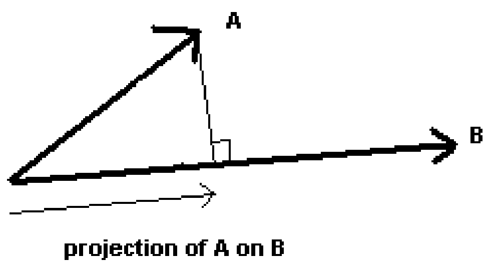
$\mathbf{A} \bullet \mathbf{B} = AB \cos \theta_{AB}$



If A and B are in same direction, $\theta = 0$
$\mathbf{A} \bullet \mathbf{B} = AB$
If A and B are perpendicular (orthogonal)
$\mathbf{A} \bullet \mathbf{B} = 0$ ← This is a common critical test.

Rules: $\mathbf{A \bullet B = B \bullet A}$; $\mathbf{A \bullet (B+C) = A \bullet B + A \bullet C}$

Projection of A on B:
$(\mathbf{A} \bullet \mathbf{B}) / B = A \cos\theta$



(c) Vector or Cross Product
$\mathbf{A} \times \mathbf{B} = \mathbf{n}\, AB \sin\theta_{AB}$
= (demonstrate cross product determinant)

Right-hand rule (direction of cross product): Point fingers of right hand in direction of A. Rotate them until they lie on B. Thumb points in direction of **AxB (n)**

Magnitude of the cross product = area of the parallelogram defined by **AxB**

If two vectors are parallel, their cross product = 0.
If one of two vectors is unreasonably small, the cross product will be zero. "Ill-conditioned vectors"

RULES:
AxB = -BxA
Ax(B+C) = AxB + AxC

AxA = 0

COORDINATE SYSTEMS

Why they are needed? Many problems are easier to solve in a specific coordinate system.

Examples: Linear antennas are best solved in cylindrical system
Point charges, which result in out-going spherical waves, are best solved in spherical system.

ORTHOGONAL COORDINATE SYSTEM

Unit vectors are all perpendicular, and form a complete set to define all of 3D space. What would happen if they were not orthogonal? Suppose we have two x-vectors and no y-vector , for instance. Then we cannot define a particular vector if it has a y-component. Furthermore, if we are at a given point, there is no unique solution for the vector definition of that point.

CARTESIAN COORDINATE SYSTEM (XYZ)

See Figure 3-8 (transparency)
Unit vectors: **x,y,z**
We will need dl,dS,dv for line,surface, and volume integrals to solve Maxwell's and related equations.
dl = dx **x** + dy **y +** dz **z**
dS = dydz **x** + dxdz **y** + dxdy **z**
dV = dxdydz

Find the area of the top of a cube with one corner at (0,0,0) and the furthest corner at (3,4,3). This means the two corners of the top are (0,0,3) and (3,4,3).

1) Define what varies (dx,dy)
2) Define any constants (z = 3)
3) Define the surface area / vector to use:
   a. What changes on the top of the cube (x and y, but not z, so dz=0)
   b. OR What vector is perpendicular to the surface being integrated? **Z**
4) Do the integration

$$A = \int_{x=0}^{3}\int_{y=0}^{4} dxdy = (3)(4)$$

Integrate the equation F = $7xyz^2$ over the top of the same cube.

$$G = \int_{x=0}^{3}\int_{y=0}^{4} (7xyz^2)dxdy = 7\left.\frac{x^2}{2}\right|_{x=0}^{3}\left.\frac{y^2}{2}\right|_{y=0}^{4}\left.z^2\right|_{z=3} = (7)\left(\frac{9}{2}\right)\left(\frac{16}{2}\right)(9)$$

CYLINDRICAL COORDINATE SYSTEM (r or ρ, ϕ, z)

See Figure 3-9 (transparency)
Unit Vectors: **r, ϕ, z**
dl = dr **r** + rdϕ **ϕ +** dz **z =** dl_r **r** + dl_ϕ **ϕ +** dl_z **z**
dS = dl_ϕ dl_z **r** + dl_r dl_z **dϕ** + dl_r dl_ϕ **z =** rdϕdz **r** + drdz **dϕ** + rdrdϕ **z**
dV = r dr dϕ dz

SPHERICAL COORDINATE SYSTEM (r, θ, ϕ)
See Figure 3-10 (transparency)
Unit Vectors: **R, θ, ϕ**
dl = dR **R** + Rdθ **θ +** R sinθ dϕ **ϕ**
dS =$dl_\phi dl_\theta$ **R** + dl_R dl_θ **ϕ** + dl_R dl_ϕ **θ**
= Rdϕ R sinθ dθ **R** + dR R sinθ dθ **ϕ** + dR Rsinθ dϕ **θ**
dV = dl_r $dl_\phi dl_\theta$ **=** dR (Rdϕ) (R sinθ dϕ)

Be able to:
1) Define the line and surface vectors, and the volume.
2) Compute the circumference, surface area,volume in any coordinate system.
   C = integral (dl) ; A = integral (dS) ; V = integral (dV)
3) Be able to integrate any function in line, surface or volume in any coordinate system

Steps:
1) Define what changes and what is constant (on the region being integrated)
2) Determine what part of dl, dS, dV to use.
3) Apply the integral. Limits are determined by the boundaries of the integration.